A. Introduction

This project is to be an individual effort for each student. All work must be your own. Any questions about the project should be addressed to Dr. Beale.

A linearized model for two degree-of-freedom motion of a helicopter in the vertical plane can be expressed by the state equations

\[ \dot{x} = Ax + Bu \]  

where \( x \in \mathbb{R}^4 \) is the state vector and \( u \) is the scalar control signal, with the following definitions:

- \( x_1 = \theta \) is the pitch angle (radians);
- \( x_2 = q \) is the pitch rate (radians per second);
- \( x_3 = u_0 \) is the horizontal velocity (meters per second);
- \( x_4 = x \) is the horizontal position (meters);
- \( u = \delta \) is the rotor thrust angle (radians).

The \( A \) and \( B \) matrices are given by

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -0.415 & -0.011 & 0 \\
9.8 & -1.43 & -0.02 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
6.27 \\
9.8 \\
0 \\
\end{bmatrix} 
\]  

Values for all matrix elements are correct for the dimensions of the state and input variables defined above.

The desired value for each state variable is zero. Starting from a non-zero initial state, the settling time for the system’s response must be less than or equal to 80 seconds. At the settling time, the state and control variables must be within the tolerances shown below.

\[
| x(t_f) | \leq \begin{bmatrix}
0.005 \text{ deg} \\
0.001 \text{ deg/s} \\
0.005 \text{ m/s} \\
0.05 \text{ m} \\
\end{bmatrix}, \quad | u(t_f) | \leq 0.001 \text{ deg} \]  

The initial conditions for the state variables and the maximum allowed magnitudes for the state and control variables are shown below. Note that units of degrees in (3) and (4) must be converted to radians before use.

\[
x(0) = \begin{bmatrix}
15 \text{ deg} \\
3 \text{ deg/s} \\
2 \text{ m/s} \\
50 \text{ m} \\
\end{bmatrix}, \quad |x(t)|_{\text{max}} = \begin{bmatrix}
17 \text{ deg} \\
16 \text{ deg/s} \\
5 \text{ m/s} \\
60 \text{ m} \\
\end{bmatrix}, \quad |u(t)|_{\text{max}} = 8.5 \text{ deg} \]
B. Tasks to be Performed

1. Design an optimal control for the given system such that all performance specifications and constraints are satisfied. You may assume that all states are measurable. If you wish, you may convert the system to discrete time and consider this as a discrete-time optimal control problem rather than as a continuous-time problem. The MATLAB command “c2d” can be used for this. If this is done, an appropriate value for the sampling period \( T \) must be chosen.

2. Simulate the closed-loop response for your final design using MATLAB and/or SIMULINK. Produce plots for your final design for each of the state variables and the control variable. The plots should clearly show that all specifications have been satisfied.

3. This part of the project deals with the stability robustness of your design, that is, how much change can there be in the plant and/or controller without losing closed-loop stability. My MATLAB function “tsypkin.m” can be used for this. The zip file for this function, and my function “circle.m” can be downloaded from the project web site. Put the two functions into your MATLAB working directory. For the purpose of this project, the syntax for using the function is \( \rho = \text{tsypkin}(dcl, 1, [0 \ dcl(2:end)]) \);. The variable \( dcl \) is the continuous-time closed-loop characteristic polynomial. The function will create a figure with 3 plots on it, and will return the 3-element vector \( \rho \). A sample plot can be obtained from the project web site. The elements in the \( \rho \) vector are the maximum allowed perturbations to the coefficients of the closed-loop characteristic equation in terms of the 1-norm, 2-norm, and infinity-norm, respectively. The larger the values in \( \rho \), the more robust your control system is to changes in the coefficients. Determine the value of \( \rho \) for both your initial design and final design.

4. Document your activities in a final report. Include plots of each state variable and the control variable. Also include a printout of the MATLAB code used to design and evaluate your controller, or provide the files electronically. Describe your design procedure and evaluate your results in terms of the specifications. Include the number of iterations in your design that were required to satisfy the specifications. Give the reasons why you chose the particular performance index you used, and discuss other cost functions which may be applicable for this problem, if any. Discuss the robustness of your design in terms of the results in Task 3, including the final figure from the robustness analysis.