The purpose of this project is to investigate the transient behavior of some simple LTI systems. In this project you will learn to create pole-zero plots and relate those to the response time of the system.

Your report for this project will consist of all the analytical (i.e., pencil/paper) work, Matlab plots and code, and relevant explanations. A list of guidelines for preparing the lab report is be posted on the ECE 220 website. Each student must do his or her own work on this project, however you may ask other students or any of the teaching staff for advice. As stated in the guidelines given in the ECE 220 course information packet, you should identify any students you talk to about the project.

1 Prelab

Before starting on the lab exercises below, reread Sections 2.6-2.7 in Linear Systems and Signals by B.P. Lathi. Pay close attention to the discussion of poles and the definition of rise time.

2 First-order system

Consider the first-order causal LTI system defined by the transfer function $H(s)$ given below:

$$H(s) = \frac{a}{s + a}.$$  

Recall that the transfer function is the Laplace transform of the impulse response $h(t)$.

(a) Determine an analytical expression for the impulse response of this system.

(b) For what values of $a$ is the impulse response guaranteed to be real? For what values of $a$ is this system BIBO stable?

(c) How is the rise time of a system defined? For this first-order system, how does the parameter $a$ affect the rise time? In this part, you should make analytical predictions. In the next part, you will confirm those predictions with Matlab.

(d) Suppose we define $P(s)$ and $Q(s)$ as the numerator and denominator terms of the transfer function, i.e.,

$$H(s) = \frac{P(s)}{Q(s)}.$$  

The poles of the system are defined as the roots of $Q(s)$ and the zeros of the system are defined as the roots of $P(s)$. Analytically determine where the pole is located for this first order system.

The Matlab command `pzplot` can be used to make plots of the poles and zeros of transfer functions. Use `help pzplot` to read more about how this command works. Then use the command to plot the pole/zero locations for this first order system when $a$ takes on the following values: 1, 10, 100, 1000.
Notes on pzplot:
- `pzplot` uses the transfer function defined by the `b` and `a` vectors and the `tf` command. It works similarly to the `lsim` command you used in Lab 2.
- In earlier versions of Matlab, the command to make pole-zero plots was `pzmap`.
- `pzplot` and `pzmap` are in the Control Systems Toolbox of Matlab. The university computers have this toolbox.

(c) Using `lsim` or `step`, make plots for the step response of this system for four different values of the `a` parameter: 1, 10, 100, 1000. Determine the rise time from these plots. Do your results agree with your predictions from part d? Why or why not?

(f) Suppose that the input to this system is a square wave with a frequency of 2 cycles per second. Generate this square wave using the following commands:

```matlab
t=0:.0001:2;
x=square(2*pi*2*t);
```

Make a plot of the square wave using the command `plot(t,x)`. If this square wave is the input to this first order system, what do you predict the output will look like? You do not need to solve for an analytical expression here, but you should be able to get an intuitive feel for what the output looks like based on the step response and your understanding of the convolution operation. Describe what you think the output will be when the parameter `a` takes on the 4 values (1, 10, 100, 1000) you used previously.

Using `lsim` simulate the output of the system for 4 different values of `a` (1, 10, 100, 1000) when the input is the square wave. Plot the input and the output on the same graph for each of these 4 cases. You should either plot them all in the same plot window or use the `subplot` command to create 4 graphs on the same page. (Note that you might want to use `orient tall` when printing the 4 subplots to get a better aspect ratio.) What do you observe about the responses for these 4 cases? Do your results agree with your predictions?

(g) Read Section 2.7-6 of the textbook. Suppose your goal is to use these square wave pulses to communicate information and that the communication signal had to pass through this first order system. Does the parameter `a` affect how fast you can transmit square pulses through this system? If so how? (You can try square waves of different frequencies simply by using `square(2*pi*f*t)`; where `f` is the desired frequency in cycles per second.)

3 Analysis of a second-order system

Consider the second-order causal LTI system defined by the transfer function $H(s)$ given below:

$$H(s) = \frac{d}{(s + c)^2 + d^2}.$$

(a) Analytically determine the pole locations of this system.

(b) For what values of the `c` and `d` parameters will this system be BIBO stable?
(c) Analytically determine the impulse response of this system.

(d) Using Matlab, explore how the $c$ and $d$ parameters affect the step response and the rise time of the system. For each of the cases you consider, make a pole-zero plot and a plot of the step response. Your report should include a thorough explanation of the cases you explored and your conclusions about the shape of the step response and the rise time.