

ECE 220 SIGNALS & SYSTEMS I
Laboratory Project 6
Fall 2006

Time allotted: 3 weeks

Report Due: 1 week after lab session completed

The purpose of this lab is to investigate the behavior of a simple second order system that is a model of an automobile suspension system. Your report for this project will consist of all the analytical work, Matlab plots and code, and relevant explanations.

A list of guidelines for preparing the lab report is posted on the ECE 220 website. Each student must do his or her own work on this project, however you may ask other students or any of the teaching staff for advice. As stated in the guidelines given in the ECE 220 course information packet, you should identify any students you talk to about the project.

1 Background

Consider the simple 1-dimensional model of an automobile suspension system shown in Figure P1.8-3 on page 149 of the textbook (*Linear Systems and Signals* by B. P. Lathi. This simplified model satisfies the second order differential equation given below:

$$\frac{d^2y(t)}{dt^2} + \left(\frac{B}{M}\right) \frac{dy(t)}{dt} + \left(\frac{K}{M}\right) y(t) = \left(\frac{B}{M}\right) \frac{dx(t)}{dt} + \left(\frac{K}{M}\right) x(t), \quad (1)$$

where K is the spring constant and B is the viscosity associated with the dashpot. (Read Section 1.8-2 of Lathi's book to learn more about the basic components of this mechanical system.) This system fits the general form of a second order system discussed in the textbook:

$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n \frac{dy(t)}{dt} + \omega_n^2 y(t) = 2\omega_n\zeta \frac{dx(t)}{dt} + \omega_n^2 x(t), \quad (2)$$

where

$$\omega_n = \sqrt{\frac{K}{M}} \quad \zeta = \frac{B}{2\sqrt{KM}}.$$

In this project you will explore how the values of the spring constant K and the dashpot viscosity B affect the performance of the system. Auto suspension systems are usually designed for a known vehicle weight. For the purpose of this project, you will assume that $M = 1$.

2 Analytical Problems

- (a) Determine an analytical expression for the frequency response $H(j\omega)$ for the system defined by Equation 2. What is the value of $H(j\omega)$ when $\omega = 0$?
- (b) Determine the location of the poles and zeros of this system in terms of ω_n and ζ . What type of frequency response characteristic do you expect this system to have? Will it act as a lowpass filter? a highpass filter? a bandpass filter?

3 Basic Problems

- (a) For this part you will assume that $\zeta = 1/\sqrt{2}$ and you will vary the value of K (hence you will vary ω_n) and observe how the system characteristics change.
- Consider 3 values of K : 0.09, 1.0, and 4.0. Use Matlab's `bode` command to obtain the Bode plot (both magnitude and phase) for each of these values of K . Note that you can use

```
[Hmag, Hphase] = bode(sys, w);
```

to store the Bode plot values in the two vectors `Hmag` and `Hphase`. The vector `w` specifies the set of ω values you want Matlab to compute the plot for. You can use `w=logspace(-2, 2, 100)` to tell Matlab to generate the Bode plot for ω values between 10^{-2} and 10^2 . Compute the Bode plots for all 3 values of K and plot them in the same window using different linestyles. Use the `legend` command to label your plot appropriately.
 - Plot the step response of the system for each value of K (0.09, 1.0, and 4.0). You should plot it out to time $t = 30$. How does the rise time of the system change as a function of ω_n ?
 - What type of filter is this system? How does ω_n affect the frequency response? How is the rise-time of the system related to ω_n ? Would you expect a shock absorbing system with small ω_n to react quickly or slowly to rapid changes in the input $x(t)$ (i.e., the road height)?
- (b) In this part you will assume that ω_n is fixed at 1.0 and you will vary the value of the damping ratio ζ . You will vary the viscosity B (which varies ζ) while holding ω_n fixed.
- Consider 4 values of B : 2, $2/\sqrt{2}$, $1/2$, and $1/3$. Compute and plot the frequency response magnitude and phase for these 4 systems using `bode`. Remember to keep $\omega_n = 1$. How does the value of $|H(j\omega_n)|$ (the magnitude of the frequency response at ω_n) change as the viscosity B varies? Does the value of B significantly affect the frequency response at frequencies far from ω_n ?
 - For each of the values B (2, $2/\sqrt{2}$, $1/2$, and $1/3$), plot the step response out to time $t = 30$. How do the oscillations and overshoot in the step response vary with B ? (Overshoot is when the step response exceeds the steady state value.)
 - Using Matlab, determine an approximate value of B for which the step response rises fastest without any overshoot. What is the corresponding value of ζ ?

4 Advanced Problems

For the automobile suspension system described above, the center of mass of the vehicle is located at a fixed offset from $y(t)$, e.g.,

$$\text{center of mass} = z(t) = y(t) + y_{\text{offset}}.$$

This simply represents the fact that the main body of the car is higher than the tires and suspension system. Note that the action of the car's suspension system is constrained by several factors:

- The center of mass should be higher than the ground, i.e., $z(t) > x(t)$. (The system model becomes invalid if the car crashes into the pavement!)

- The suspension system has limited motion in the vertical dimension, *i.e.*, $y(t)$ cannot be arbitrarily large or the spring/dashpot system will come apart. (You can only stretch a shock absorber so far!) This means that

$$|y(t) - x(t)| < y_{\text{limit}}.$$

For simplicity in this problem you will assume that $M = 1$, $y_{\text{offset}} = 0.5$, and $y_{\text{limit}} = 0.5$.

- (a) Load the file `lab6data.mat`. This file contains two vectors, \mathbf{x} and \mathbf{t} . \mathbf{x} represents the level of the road surface at the time samples contained in \mathbf{t} . Plot \mathbf{x} versus \mathbf{t} . Observe that the road surface consists of a trapezoidal surface plus some small scale fluctuations. You can think of the trapezoid as representing a section of road that is higher than the rest.

In this part, you should determine a set of parameters K and B so that the suspension system smoothes out the bumps in the road surface without violating the constraints noted above. Once you have your values of K and B , you can compute the output $y(t)$ (and hence $z(t)$) for your choice of parameters using the `lsim` command with \mathbf{x} as the input.

Plot your results for several values of K and B . Compare the relative merits of each of these sets of parameters.

- (b) Suspension systems are typically designed with a specific mass in mind. For instance, we have assumed $M = 1$ for this project. What happens if someone loads up their car with several boxes of ECE 220 textbooks? The mass will obviously change. How much can M vary from its nominal value of 1 before the system constraints are violated? For the best values of K and B you found in the previous part, what is the range of values of M for which the system will still obey the constraints?
- (c) Write a short memo describing how you would choose the K and B parameters to give the car's passengers the best ride. This memo should summarize what you've learned from this project. Your memo should be concise: 1 page (or less) of text, plus 1 page (or less) of figures. It should provide your busy boss Mr. Ratbert everything that he needs to know to design a better auto suspension system.