Problem Set 5
Fall 2001

Issued: Tuesday, October 23, 2001
Due: Wednesday, October 31, 2001

Reading in Porat
Week of 10/22/01 — Chapter 5, omitting section 5.7.2

Problem 5.1
Suppose you need to implement the linear convolution of a 10,000-point sequence with an FIR impulse response that is 100 points long. To implement this convolution, you have been told that you must use 256-point DFT’s and inverse DFT’s.

(a) If you use the overlap-add method, what is the minimum number of 256-point DFT’s and the minimum number of 256-point inverse DFT’s you have to compute in order to implement the convolution for the entire 10,000-point sequence. Justify your answer.

(b) If you use the overlap-save method, what is the minimum number of 256-point DFT’s and the minimum number of 256-point inverse DFT’s you have to compute in order to implement the convolution for the entire 10,000-point sequence. Justify your answer.

(c) For the 10,000-point sequence and 100-point filter, compare the number of arithmetic operations (multiplies and adds) required in the overlap-add method, overlap-save method, and direct convolution.

Problem 5.2
The butterfly in Figure 1 below was taken from the flowgraph for a 16-point decimation-in-frequency FFT, where the input sequence was arranged in normal order. A 16-point decimation-in-frequency FFT will have four stages, indexed $m = 1, \ldots, 4$. Which of the four stages have butterflies of this form? Justify your answer.

![Figure 1: Butterfly for Problem 5.2](image-url)
Problem 5.3
The purpose of this problem is to encourage you to review the derivations of the decimation-in-frequency and decimation-in-time algorithms and to prepare you for the Matlab assignment.

(a) Sketch a flowgraph implementation of an 8-point decimation-in-frequency FFT. Compute the values of each twiddle factor on your flowgraph, i.e., \( W_4^{-2} = e^{-j2\pi/4} = e^{-j\pi} = -1 \)

(b) Sketch a flowgraph implementation of an 8-point decimation-in-time FFT. Compute the values of each twiddle factor on your flowgraph, as you did for part (a).

(c) Assume that your input sequence is \( x[n] = 1 \) for \( n = 0, \ldots, 7 \). Compute the values you would see at each node of the decimation-in-frequency flowgraph.

(d) Repeat part (c) for the decimation-in-time flowgraph.

Problem 5.4
Each part of this problem may be solved independently. All parts use the signal \( x[n] \) shown in Figure 2.

(a) Compute \( X_4[k] \), the 4-point DFT of \( x[n] \).

(b) Let \( X_5[k] \) be the five-point DFT of \( x[n] \), and let \( H[k] \) be the five-point DFT of the impulse response \( h[n] \) shown in Figure 3. Let \( Y[k] = H[k]X_5[k] \) for \( 0 \leq k \leq 4 \). Sketch \( y[n] \), the five-point inverse DFT of \( Y[k] \).