Problem ECE410-6
We wish to design an FIR lowpass filter satisfying the specifications

\[ 0.9 \leq |H(e^{j\omega})| \leq 1.1, \quad 0 \leq |\omega| \leq 0.4\pi \]
\[ |H(e^{j\omega})| \leq 0.06, \quad 0.6\pi \leq |\omega| \leq \pi \]

by applying a window \( w[n] \) to the impulse response \( h_d[n] \) for the ideal discrete-time lowpass filter with cutoff \( \omega_c = 0.5\pi \). Which of the windows listed in Section 7.21 of Oppenheim and Schafer can be used to meet this specification? For each window that you claim will satisfy this specification, give the minimum length \( M + 1 \) required for the filter.
Susan designed two DT filters using the CT prototype filter with the frequency response shown in Figure 1. She designed Filter A using the impulse invariance method. She designed Filter B using the bilinear transformation method. Susan used the design parameter $T_d = 2$ for both of these methods. Note that the frequency response of the CT prototype filter is between 0.9 and 1.0 in the passband and 0 and 0.1 in the stopband. These tolerances are indicated by the dashed lines in Figure 1.

Unfortunately, after Susan completed the designs, her computer crashed. She had saved the two sets of filter coefficients onto a disk, but they got mixed in with the coefficients for two other filters. Thus she has 4 filters (numbered 1 through 4). Susan plotted the frequency response magnitudes for these 4 filters using `freqz`. These frequency responses are shown in Figure 2. Can you help Susan decide which two filters correspond to her impulse invariance design (Filter A) and her bilinear transformation design (Filter B)?

For each of the 4 filters, indicate whether it corresponds to the impulse invariance design ($T_d = 2$) or the bilinear transformation design ($T_d = 2$) or neither. **Justify your answers!**

Summarize your results using the form on the last page.

**Figure 1:** Frequency response of the CT prototype filter used in Problem 7.
Figure 2: Frequency response magnitudes for filters 1-4 in Problem 7. The dashed lines indicate the 0.9 to 1.0 tolerance for the passband and the 0 to 0.1 tolerance for the stopband.
Filter 1: impulse invariance design? bilinear transformation design? neither?
Justification:

Filter 2: impulse invariance design? bilinear transformation design? neither?
Justification:

Filter 3: impulse invariance design? bilinear transformation design? neither?
Justification:

Filter 4: impulse invariance design? bilinear transformation design? neither?
Justification: