Group delay

Group delay is a useful measure of phase linearity:

\[ \text{grd} = -\frac{d}{d\omega} \{ \arg(H(e^{j\omega})) \} \]

If phase is linear, then group delay is constant

Concept of group delay relates to effect of phase response on narrowband (NB) signals

NB signal: \( x[n] = s[n] \cos(\omega_0 n) \) (recall sketch from last time)

Assume \( x[n] \) is input to a real DT LTI system and that phase obeys a linear approximation around \( \omega_0 \), i.e.,

\[ \angle H(e^{j\omega}) \approx -\phi_0 - \omega n_d \]

Then can show (see O/S/B Problem 5.57)

\[ y[n] \approx |H(e^{j\omega_0})| s[n - n_d] \cos(\omega_0 (n - n_d) - \phi_0) \]

Group delay example: FIR filter characteristics

33-point FIR filter \( h[n] \)

Frequency response magnitude →

Group delay →
Group delay example: 2 input signals

Consider 2 narrowband pulses, $x_1[n]$ and $x_2[n]$:

- **Input signal:** $x_1[n]$
- **Input signal:** $x_2[n]$

Fourier transform magnitudes, $|X_1(e^{j\omega})|$ and $|X_2(e^{j\omega})|$.

Group delay example: input/output of FIR filter

Compare the inputs/outputs of the FIR filter

- **FIR filter:** Input/Output signals: $x_1[n]$ and $y_1[n]$
- **FIR filter:** Input/Output signals: $x_2[n]$ and $y_2[n]$

Are the signals delayed by the right amounts?
Group delay example: 8th-order IIR filter characteristics

8th-order IIR filter $h[n]$

Frequency response magnitude

Group delay

Group delay example: input/output of IIR filter

Compare the inputs/outputs of the IIR filter

Are the signals delayed by the right amounts?
Consider a wideband pulse

Time signal and frequency response magnitude are shown below:

Pulse $x[n]$ →

Frequency response of pulse $|X(e^{j\omega})|$ →

If this signal is input to FIR and IIR filters, what does output look like?

Wideband pulse output of FIR/IIR filters

Why does IIR pulse look so distorted?
Application: filtering noisy pulses

Linear phase filtering is often preferable

![Input signal pnd₁](image)

![Filtered versions of pnd₁](image)

Final comments on group delay

Causal filters can’t have zero phase (see Section 5.7 of O/S/B)

Easy to guarantee generalized linear phase with FIR filters
- Satisfy symmetry properties
- Four types (even/odd lengths, symmetric and antisymmetric)

Can we have non-integer group delay? If so, what does it mean? See Section 4.5 of O/S/B.
Pole-zero plot and frequency response of IIR filter

Pole-zero plot of IIR filter

Frequency response of IIR filter

Pole-zero plot and frequency response of FIR filter

Pole-zero plot of FIR filter

Frequency response of FIR filter
In-class problem

Consider the 2 FIR filters defined below:

\[ h_1[n] = \frac{1}{2} \delta[n] - \frac{1}{2} \delta[n - 1] \]

\[ h_2[n] = \frac{1}{2} \delta[n] - \frac{1}{2} \delta[n - 2] \]

Determine the system function \( H(z) \) for each filter and sketch its frequency response magnitude.