

**Problem 5** (20 points)

Susan designed two DT filters using the CT prototype filter with the frequency response shown in Figure 5.1. She designed Filter A using the impulse invariance method. She designed Filter B using the bilinear transformation method. Susan used the design parameter  $T_d = 2$  for both of these methods. Note that the frequency response of the CT prototype filter is between 0.9 and 1.0 in the passband and 0 and 0.1 in the stopband. These tolerances are indicated by the dashed lines in Figure 5.1.

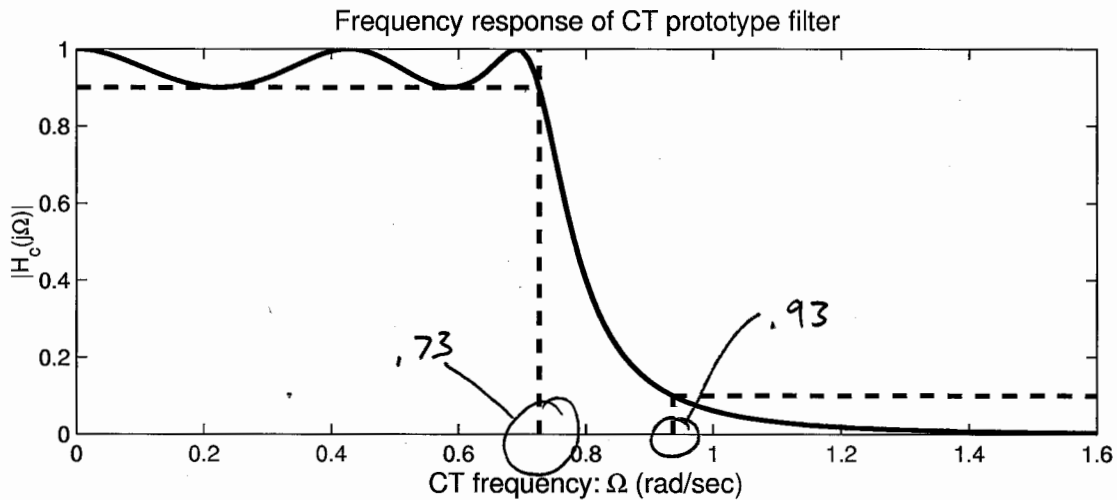


Figure 5.1: Frequency response of the CT prototype filter used in Problem 5.

Unfortunately, after Susan completed the designs, her computer crashed. She had saved the two sets of filter coefficients onto a disk, but they got mixed in with the coefficients for two other filters. Thus she has 4 filters (numbered 1 through 4). Susan plotted the frequency response magnitudes for these 4 filters using `freqz`. These frequency responses are shown in Figure 5.2. Can you help Susan decide which two filters correspond to her impulse invariance design (Filter A) and her bilinear transformation design (Filter B)?

For each of the 4 filters, indicate whether it corresponds to the impulse invariance design ( $T_d = 2$ ) or the bilinear transformation design ( $T_d = 2$ ) or neither. **Justify your answers!**

Summarize your results on page 12.

FROM FIG. 5.1:

$$\Omega_p \approx 0.73 \text{ rad/sec}$$

$$\Omega_s \approx 0.93 \text{ rad/sec}$$

IMP. INVARIANCE MAPPING:

$$\omega_p = \Omega_p T_d = (0.73)2 = 1.46 \approx 0.47\pi$$

$$\omega_s = \Omega_s T_d = (0.93)2 = 1.86 \approx 0.6\pi$$

BILINEAR TRANS. MAPPING:

$$\omega_p = 2 \tan^{-1} \left( \frac{\Omega_p T_d}{2} \right) = 2 \tan^{-1} (0.73) = 1.26 \approx 0.4\pi$$

$$\omega_s = 2 \tan^{-1} \left( \frac{\Omega_s T_d}{2} \right) = 2 \tan^{-1} (0.93) = 1.5 \approx 0.48\pi$$

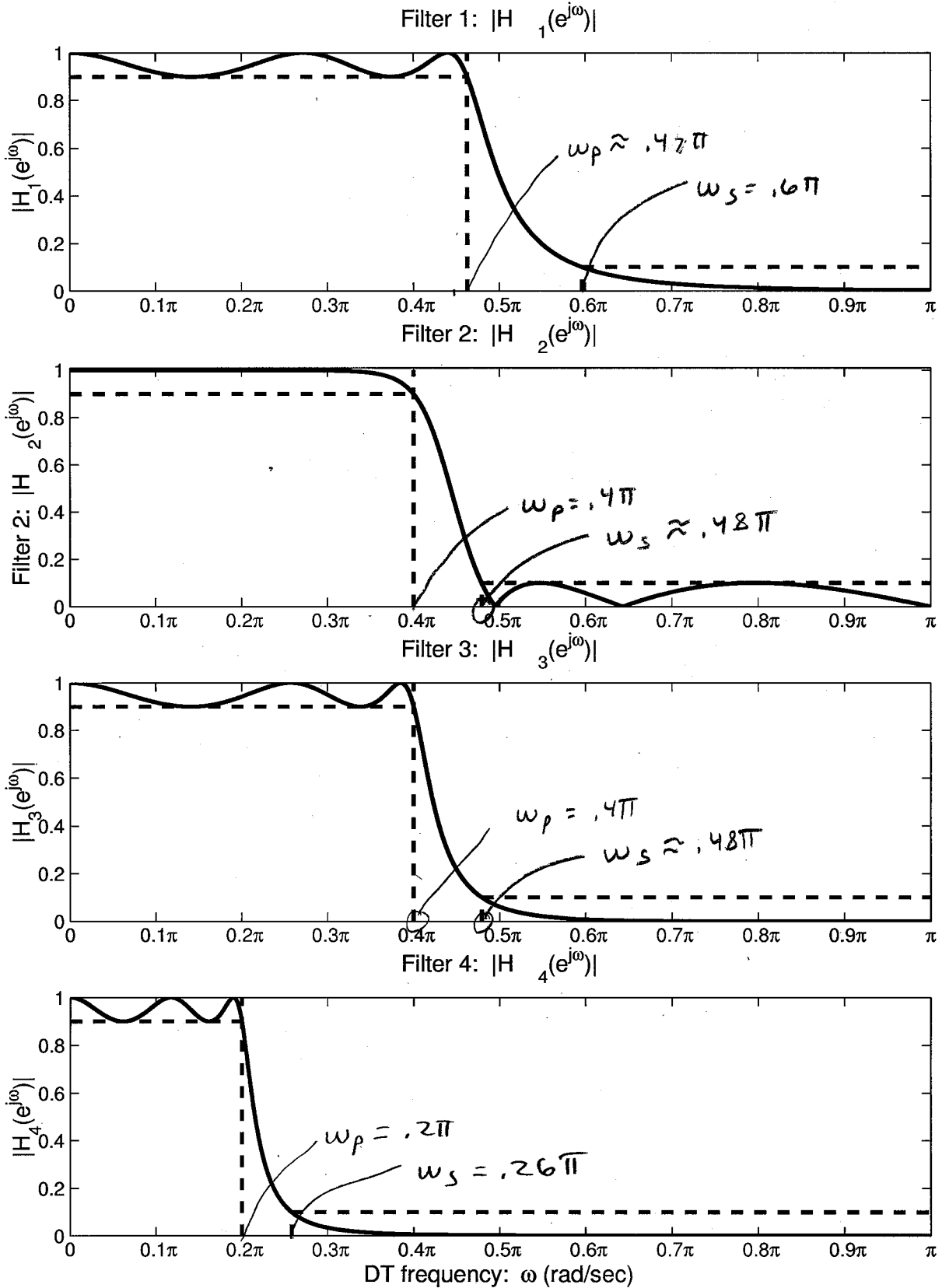


Figure 5.2: Frequency response magnitudes for filters 1-4 in Problem 5. The dashed lines indicate the 0.9 to 1.0 tolerance for the passband and the 0 to 0.1 tolerance for the stopband.

Filter 1: impulse invariance design?

bilinear transformation design?

neither?

Justification:

$$\left. \begin{array}{l} \omega_p \approx .47\pi \\ \omega_s \approx .6\pi \end{array} \right\} \text{FROM PLOT}$$

THESE MATCH PASS + STOP BAND EDGES WE EXPECT

FOR IMP. INVARIANCE. FILTER LOOKS LIKE A COPY  
OF PROTOTYPE W/ FREQ. SCALING  $\omega = \Omega T_D \Rightarrow$  IMP. INVARIANCE.

Filter 2: impulse invariance design?

bilinear transformation design?

neither?

Justification:

PASS + STOP BAND EDGES ARE  $.4\pi$  +  $.48\pi$  RESPECTIVELY  
 $\rightarrow$  THESE MATCH B.L.T. DESIGN.

BUT: B.L.T. MAPPING CANNOT GET RID OF RIPPLES  
IN PASSBAND + ADD THEM IN STOPBAND  
 $\Rightarrow$  NEITHER

Filter 3: impulse invariance design?

bilinear transformation design?

neither?

Justification:

$$\left. \begin{array}{l} \omega_p \approx .4\pi \\ \omega_s \approx .48\pi \end{array} \right\} \text{MATCHES B.L.T. PREDICTIONS}$$

PLUS  $|H_3(e^{j\omega})|$  LOOKS LIKE CORRECT WARPING OF  
PROTOTYPE (RIPPLES IN PASSBAND MATCH, PLUS  
NO RIPPLES IN STOPBAND).

Filter 4: impulse invariance design?

bilinear transformation design?

neither?

Justification:

$$\left. \begin{array}{l} \omega_p = .2\pi \\ \omega_s = .26\pi \end{array} \right\} \leftarrow \text{DON'T MATCH ANY OF OUR PREDICTIONS FOR IMP. INV. OR B.L.T. DESIGNS.}$$

 $\Rightarrow$  NEITHER