RAT: 11/1/05

1. Name: ____________________________
2. T/F In window design, the filter is defined as the multiplication of the desired impulse response and a finite-duration window, *i.e.*, $h[n] = h_d[n]w[n]$.
3. *This question was ambiguous and has been removed.*
4. T/F One of the commonly-used windows described in this section is the Fourier window.
5. T/F In window filter design, the width of the transition band is primarily determined by the height of the peak sidelobe of the Fourier transform of the window.

Frequency-selective filtering

Design Problem:

> Specify $H(e^{j\omega})$, $h[n]$ or difference equation such that system passes a set of frequencies and rejects others

Types of LTI filters:

- Infinite Impulse Response (IIR), *i.e.,* infinite length
- Finite Impulse Response (FIR), *i.e.,* finite length

Compare IIR and FIR ⇒

Filter design choices: IIR vs. FIR

IIR

$$y[n] = \sum_{k=0}^{N} a_k y[n-k] + \sum_{m=0}^{M} b_m x[n-m]$$

- Must implement recursively! (a_k and b_m coefficients are non-zero)
- Phase is difficult to control
- Fewer multiplies and adds for same $|H(e^{j\omega})|$
- Stability can be a problem
- Can build on analog filter design techniques (to get closed form solutions)

FIR

$$y[n] = \sum_{m=0}^{M} b_m x[n-m]$$

$$h[n] = \begin{cases} b_n & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

- Recursion not required! (only b_m coefficients are non-zero)
- Linear phase easy to obtain
- More multiplies and adds for same $|H(e^{j\omega})|$
- FIR filters always stable
- No analog history to build upon

Linear phase FIR filters ⇒

Generalized linear phase FIR filters

Why is linear phase desirable?

$\longrightarrow$ constant group delay, no pulse dispersion

Four types: (see O/S/B section 5.7)

$h[n]$ is nonzero for $0 \le n \le M \longrightarrow$ length $= M + 1$

Type	Symmetry	M	Group delay	Constraints
I	$h[n] = h[M-n]$	even	$M/2$	none
II	$h[n] = h[M-n]$	odd	$M/2$	zero at $z = -1$
III	$h[n] = -h[M-n]$	even	$M/2$	zeros at $z = \pm 1$
IV	$h[n] = -h[M-n]$	odd	$M/2$	zero at $z = +1$

FIR design methods ⇒

FIR filter design methods

- Window design
  - $h[n] = w[n]h_d[n]$ (truncate impulse response of ideal prototype)
  - Matlab command: `fir1`
- Optimal minmax design
  - Parks-McClellan algorithm. See O/S/B Section 7.4.
  - Matlab command: `remez` or `firpm`
- Frequency-sampling
  - Get samples of desired response $\longrightarrow$ take inverse DFT
  - See Exercise 5.4 in Matlab Book (Buck/Daniel/Singer)
  - Matlab command: `fir2`

We'll focus on window design. Start with In-class #1 $\Rightarrow$

In-class problem #1 **Group #:**

Consider an ideal lowpass filter (LPF) with linear phase:

$$H_{\text{LP}}(e^{j\omega}) = \begin{cases} 1e^{-j\omega M/2} & |\omega| < \frac{\pi}{4} \\ 0 & \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$$

(a) Determine the impulse response $h_{\text{LP}}[n]$ of this filter.

(b) Is the impulse response symmetric? If so, what point is it symmetric about?

In-class problem #2 **Group #:**

Determine the impulse response of a 5-point causal lowpass filter using the window method. Use the ideal LPF from In-class #1 as your prototype.

(a) What should the point of symmetry of this filter be?

(b) Sketch $w[n]$, the rectangular window used for this design.

(c) Find and sketch the impulse response $h[n]$ of the 5-point filter.