1. Name: ____________________________

2. T/F In window design, the filter is defined as the multiplication of the desired impulse response and a finite-duration window, i.e.,
\[ h[n] = h_d[n]w[n]. \]

3. This question was ambiguous and has been removed.

4. T/F One of the commonly-used windows described in this section is the Fourier window.

5. T/F In window filter design, the width of the transition band is primarily determined by the height of the peak sidelobe of the Fourier transform of the window.

---

**Frequency-selective filtering**

Design Problem:

Specify \( H(e^{j\omega}) \), \( h[n] \) or difference equation such that system passes a set of frequencies and rejects others

Types of LTI filters:

- Infinite Impulse Response (IIR), i.e., infinite length

- Finite Impulse Response (FIR), i.e., finite length

Compare IIR and FIR ⇒
Filter design choices: IIR vs. FIR

\[ y[n] = \sum_{k=0}^{N} a_k y[n-k] + \sum_{m=0}^{M} b_m x[n-m] \]

- Must implement recursively! 
  \((a_k \text{ and } b_m \text{ coefficients are non-zero})\)
- Phase is difficult to control
- Fewer multiplies and adds for same \(|H(e^{j\omega})|\)
- Stability can be a problem
- Can build on analog filter design techniques (to get closed form solutions)

\[ y[n] = \sum_{m=0}^{M} b_m x[n - m] \]

\[ h[n] = \begin{cases} 
  b_n & 0 \leq n \leq M \\
  0 & \text{otherwise} 
\end{cases} \]

- Recursion not required!  
  (only \(b_m\) coefficients are non-zero)
- Linear phase easy to obtain
- More multiplies and adds for same \(|H(e^{j\omega})|\)
- FIR filters always stable
- No analog history to build upon

Linear phase FIR filters ⇒

Generalized linear phase FIR filters

Why is linear phase desirable?  
→ constant group delay, no pulse dispersion

Four types: (see O/S/B section 5.7)
\(h[n]\) is nonzero for \(0 \leq n \leq M\) → length = \(M + 1\)

<table>
<thead>
<tr>
<th>Type</th>
<th>Symmetry</th>
<th>(M)</th>
<th>Group delay</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(h[n] = h[M - n])</td>
<td>even</td>
<td>(M/2)</td>
<td>none</td>
</tr>
<tr>
<td>II</td>
<td>(h[n] = h[M - n])</td>
<td>odd</td>
<td>(M/2)</td>
<td>zero at (z = -1)</td>
</tr>
<tr>
<td>III</td>
<td>(h[n] = -h[M - n])</td>
<td>even</td>
<td>(M/2)</td>
<td>zeros at (z = \pm 1)</td>
</tr>
<tr>
<td>IV</td>
<td>(h[n] = -h[M - n])</td>
<td>odd</td>
<td>(M/2)</td>
<td>zero at (z = +1)</td>
</tr>
</tbody>
</table>

FIR design methods ⇒
FIR filter design methods

- Window design
  - \( h[n] = w[n]h_d[n] \) (truncate impulse response of ideal prototype)
  - Matlab command: `fir1`

- Optimal minmax design
  - Parks-McClellan algorithm. See O/S/B Section 7.4.
  - Matlab command: `remez` or `firpm`

- Frequency-sampling
  - Get samples of desired response \( \rightarrow \) take inverse DFT
  - See Exercise 5.4 in Matlab Book (Buck/Daniel/Singer)
  - Matlab command: `fir2`

We’ll focus on window design. Start with In-class #1 ⇒
Consider an ideal lowpass filter (LPF) with linear phase:

$$H_{LP}(e^{j\omega}) = \begin{cases} 
1e^{-j\omega M/2} & |\omega| < \frac{\pi}{4} \\
0 & \frac{\pi}{4} \leq |\omega| \leq \pi
\end{cases}$$

(a) Determine the impulse response $h_{LP}[n]$ of this filter.

(b) Is the impulse response symmetric? If so, what point is it symmetric about?
Determine the impulse response of a 5-point causal lowpass filter using the window method. Use the ideal LPF from In-class #1 as your prototype.

(a) What should the point of symmetry of this filter be?

(b) Sketch \( w[n] \), the rectangular window used for this design.

(c) Find and sketch the impulse response \( h[n] \) of the 5-point filter.