1. Name____________________

2. T/F Compared to other windows discussed in Section 7.2, the Blackman window has the narrowest mainlobe (of the Fourier transform).

3. T/F Increasing the length of a rectangular window increases the width of the mainlobe of the Fourier transform of the window.

4. T/F Compared to other windows discussed in Section 7.2, the rectangular window has the highest sidelobes.

5. T/F Kaiser developed a new method of nonlinear filtering to remove quadratic noise.

FIR design: window method

Basic idea: truncate impulse response of ideal prototype

\[ h[n] = w[n]h_d[n] \]

How does \( H(e^{j\omega}) \) relate to \( H_d(e^{j\omega}) \)?

How do we guarantee linear phase?

Matlab examples ⇒
Parameters in window design

Window length:

Window type:

Common windows ⇒

Commonly-used windows

How to get famous in DSP? Design a window!

Window frequency responses ⇒
Characteristics of commonly-used windows

From Oppenheim/Schafer/Buck Table 7.1

<table>
<thead>
<tr>
<th>Type</th>
<th>Peak SL</th>
<th>≈ ML width</th>
<th>Peak approx. error (20 \log_{10}\delta)</th>
<th>Equiv. Trans. width (Kaiser)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular Bartlett</td>
<td>-13 dB</td>
<td>(4\pi/(M + 1))</td>
<td>-21 dB</td>
<td>(1.81\pi/M)</td>
</tr>
<tr>
<td>Bartlett</td>
<td>-25 dB</td>
<td>(8\pi/M)</td>
<td>-25 dB</td>
<td>(2.37\pi/M)</td>
</tr>
<tr>
<td>Hanning</td>
<td>-31 dB</td>
<td>(8\pi/M)</td>
<td>-44 dB</td>
<td>(5.01\pi/M)</td>
</tr>
<tr>
<td>Hamming</td>
<td>-41 dB</td>
<td>(8\pi/M)</td>
<td>-53 dB</td>
<td>(6.27\pi/M)</td>
</tr>
<tr>
<td>Blackman</td>
<td>-57 dB</td>
<td>(12\pi/M)</td>
<td>-74 dB</td>
<td>(9.19\pi/M)</td>
</tr>
</tbody>
</table>

Tradeoffs in window design method:
⇒ mainlobe width versus sidelobe level
We wish to design an FIR lowpass filter satisfying the specifications

\[ 0.95 \leq |H(e^{j\omega})| \leq 1.05, \quad 0 \leq |\omega| \leq 0.25\pi \]

\[ |H(e^{j\omega})| \leq 0.1, \quad 0.35\pi \leq |\omega| \leq \pi \]

by applying a window \( w[n] \) to the impulse response \( h_d[n] \) for the ideal discrete-time lowpass filter with cutoff \( \omega_c = 0.3\pi \). Which of the windows listed in Section 7.21 of Oppenheim and Schafer can be used to meet this specification? For each window that you claim will satisfy this specification, give the minimum length \( M + 1 \) required for the filter.
Kaiser window design method

Tradeoff in ML width and SL area can be quantified by looking for a window function concentrated around $\omega = 0$ in freq. domain:

- Slepian: prolate spheroidal functions. Hard to work with!
- Kaiser: near-optimal window can be created from a zeroth order Bessel function. Easier to work with!

Kaiser window has 2 parameters: length $= M + 1$, shape $= \beta$

Kaiser developed empirical formulas to choose $M$ and $\beta$ given

$\Delta \omega = \omega_s - \omega_p$ (transition bandwidth)
$A = -20 \log_{10} \delta$ (ripple)

Note: transition bandwidth of equivalent Kaiser window often a better predictor of transition width than mainlobe width of window. What would Kaiser suggest for window lengths in-class #1?

Final remarks

Matlab program for FIR design is `fir1`

- Specify $M =$ length $- 1$, $\omega_c / \pi$, and window
- How to get $\omega_c$?

$$\omega_c = \frac{\omega_s + \omega_p}{2}$$

- Use ‘noscale’ option to avoid Matlab scaling the filter coefficients.

Other design methods:

- Frequency sampling design: see Ex. 5.4 in Matlab book.
- Parks-McClellan design:
  - Criteria: minimize the maximum approximation error
    $\rightarrow$ results in equiripple designs
  - See Section 7.4 of textbook for more info.