ECE 410 Introduction to Signal Processing

Practice Problems

December 6, 2005

Problem 1 (25 points)
Figure 1.1 shows a simple system for analyzing the frequency content of the signal $x_c(t)$. For this problem, it is known that $x_c(t) = \cos(2\pi(75)t)$. The sampling rate f_s is 500 Hz, and $w[n]$ is an 40-point rectangular window.

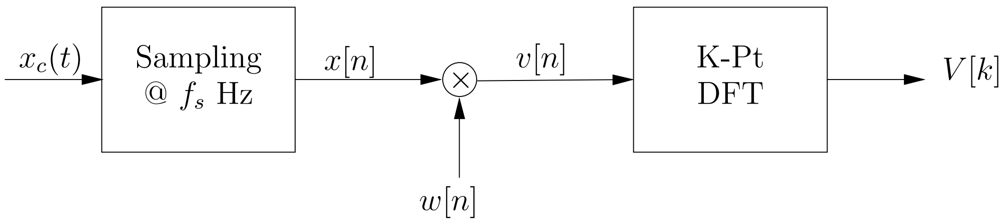

Figure 1.1: Spectral analysis system for Problem 4

1. Figure 1.2 shows 5 possible choices for the log magnitude of the Fourier transform of the windowed signal $v[n]$. (Note: $v[n]$ is the signal **prior** to the DFT operation.) Given that you know $x_c(t)$, f_s, and $w[n]$, which of the plots could be $20\log_{10}|V(e^{j\theta})|$? **Justify your answer clearly. Answers without justification will be assumed to be guesses and receive no credit.** You may find the information in Table 1.1 to be useful.

Window type	Peak sidelobe (dB)	Approximate mainlobe width
Rectangular	-13	$4\pi/N$
Bartlett	-25	$8\pi/(N-1)$
Hanning	-31	$8\pi/(N-1)$
Hamming	-41	$8\pi/(N-1)$
Blackman	-57	$12\pi/(N-1)$

Table 1.1: Parameters of common windows. Window length is defined to be N. Relative sidelobe levels are given in dB ($20\log_{10}|\cdot|$) relative to the peak of the window transform.

2. Which one of the following DFT lengths will ensure that $V[k]$ contains a sample at exactly the frequency of the sinusoid in $x_c(t)$? Justify your answer.

- 50 points
- 80 points
- 100 points

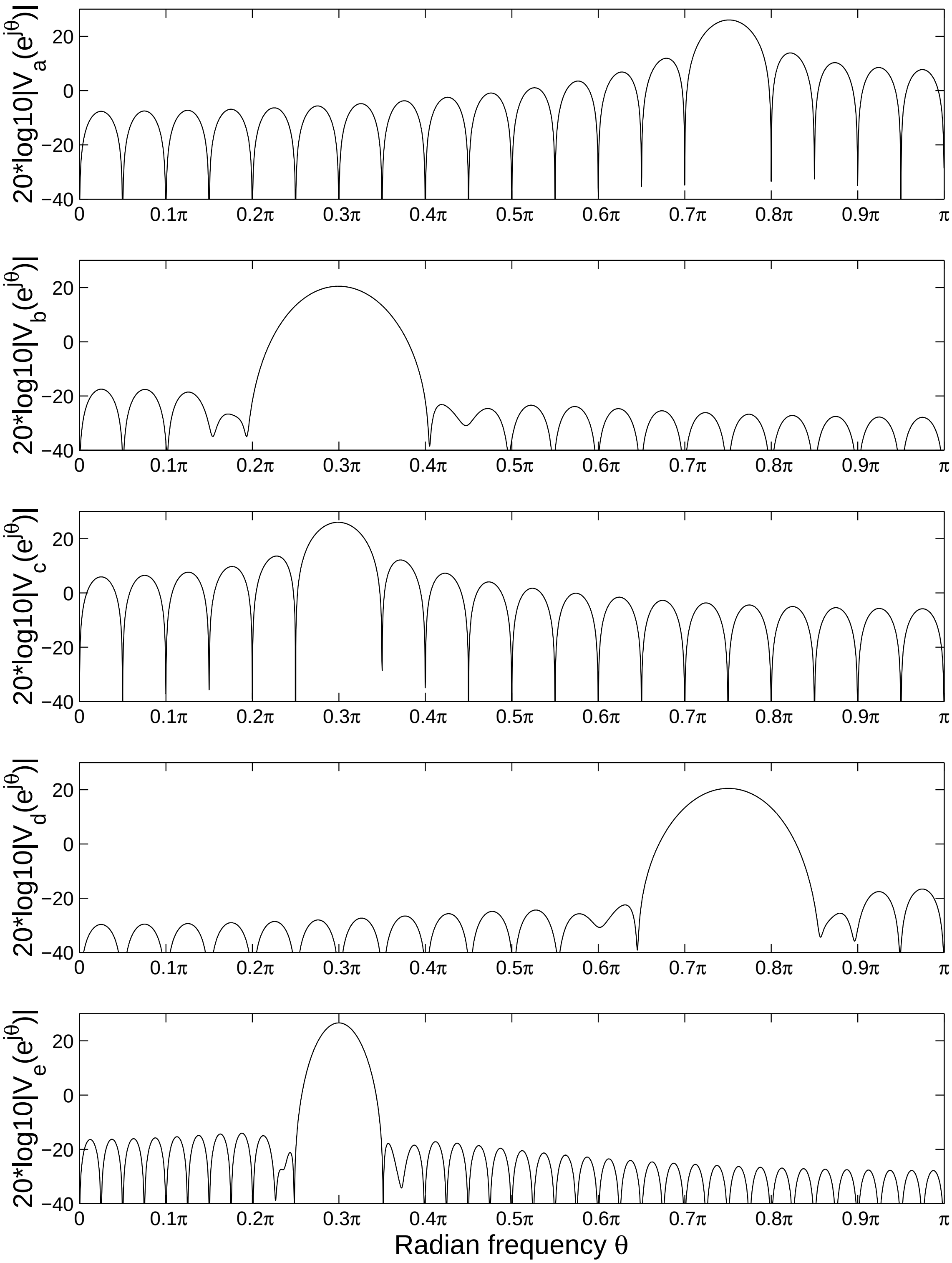

Figure 1.2: Plots of $20 \log_{10} |V_a(e^{j\theta})|$ through $20 \log_{10} |V_e(e^{j\theta})|$

Problem 2 (10 points)

This problem concerns the spectral analysis of a 128-point signal $x[n]$. This signal is windowed with a 128-point rectangular window to get the DTFT $|X_{rect}(e^{j\omega})|$ and with a 128-point Hamming window to obtain the DTFT $|X_{ham}(e^{j\omega})|$. Figure 2.1 shows both of these magnitude spectra.

Listed below are five different possible signals $x_1[n]$ through $x_5[n]$. Indicate for each of these signals if it could be the signal $x[n]$ which produced $|X_{rect}(e^{j\omega})|$ and $|X_{ham}(e^{j\omega})|$. Note that it is possible that more than one or none of the signals listed could produce the graphs given. You may find Table 2.1 useful. **Justify your answer clearly. Answers without justification will be assumed to be guesses and receive no credit.**

$$
\begin{aligned}
x_1[n] &= \sin(0.28\pi n) + 0.012\sin(0.45\pi n + \pi/2) \\
x_2[n] &= 0.5\sin(0.28\pi n) + 0.5\sin(0.281\pi n) + 0.012\sin(0.45\pi n + \pi/2) \\
x_3[n] &= \sin(0.28\pi n) + 0.001\sin(0.45\pi n + \pi/2) \\
x_4[n] &= \sin(0.28\pi n) + 0.012\sin(0.45\pi n) \\
x_5[n] &= 0.5\sin(0.28\pi n) + 0.5\sin(0.31\pi n) + 0.012\sin(0.45\pi n + \pi/2)
\end{aligned}
$$

Window Type of Length $M+1$	Approximate Width of Mainlobe	Peak Side-Lobe Amplitude (dB)
Rectangular	$4\pi/(M+1)$	-13
Hamming	$8\pi/M$	-41

Table 2.1: Commonly Used Windows

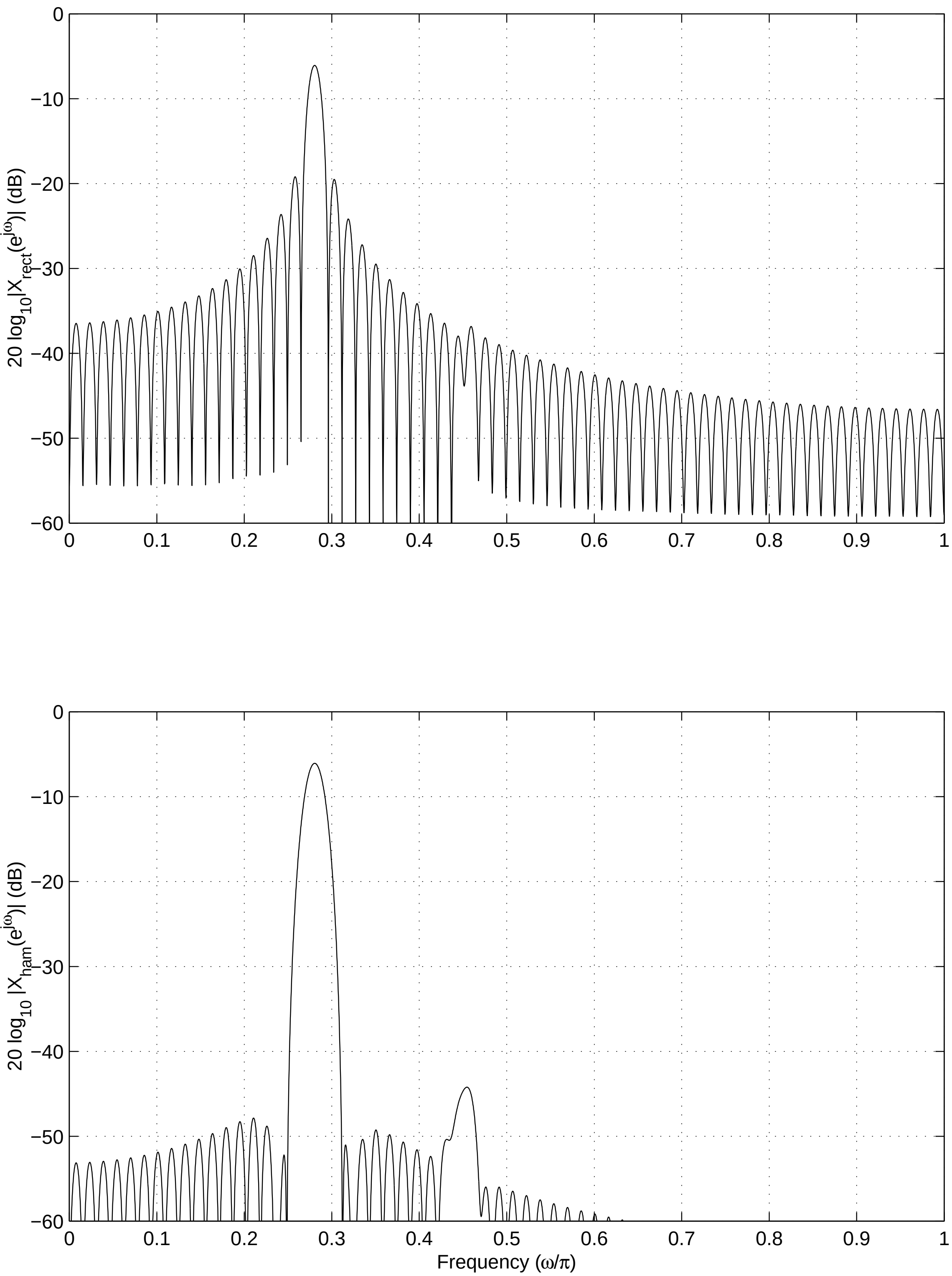

Figure 2.1: DTFT Magnitudes $|X_{rect}(e^{j\omega})|$ and $|X_{ham}(e^{j\omega})|$

Problem 3 (10 points)
The system shown in Figure 3.1 is a spectral analyzer for the continuous-time signal $x_c(t)$. The signal is sampled at a rate of 50 Hz and the resulting discrete-time signal $x[n]$ is windowed with a 201-point Hamming window. The system takes a 1000-point DFT of the windowed signal and stores the result in $V[k]$ for $k = 0, \ldots, 999$.

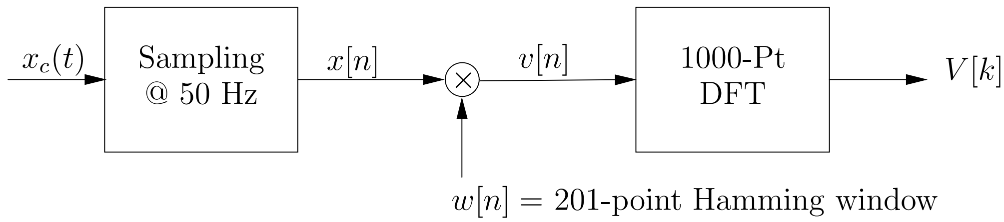

Figure 3.1:

Answer the following questions about the system. If you need information about common window properties, see page 5.

1. What is the equivalent frequency spacing in continuous-time of adjacent frequency samples in $V[k]$? In other words what is the spacing in Hz between $V[0]$ and $V[1]$?
2. To what continuous-time frequency does the index $k = 800$ in $V[k]$ correspond?
3. Suppose that $x_c(t)$ consists of two sinusoids: $x_c(t) = \cos(\Omega_0 t) + \cos((\Omega_0 + \Delta\Omega)t)$. What is the smallest value of $\Delta\Omega$ for which a plot of $|V[k]|$ will show two distinct spectral peaks? Justify your answer (indicating what, if any, approximations you have made).

Comparison of commonly used windows (from Oppenheim/Schafer/Buck, Table 7.1). Note that the window length is $M + 1$.

Type of Window	Peak Sidelobe Amplitude (dB relative to peak)	Approximate Width of Mainlobe	Peak Approximation Error $20\log_{10}\delta$ (dB)
Rectangular	-13	$\frac{4\pi}{M+1}$	-21
Bartlett	-25	$\frac{8\pi}{M}$	-25
Hanning	-31	$\frac{8\pi}{M}$	-44
Hamming	-41	$\frac{8\pi}{M}$	-53
Blackman	-57	$\frac{12\pi}{M}$	-74

Problem 4

Figure 4.1 shows the magnitude $|V[k]|$ of the 128-point DFT $V[k]$ for a signal $v[n]$. The signal $v[n]$ was obtained by multiplying $x[n]$ by a 128-point rectangular window $w[n]$, i.e. $v[n] = x[n]w[n]$. Note that Figure P3-1 shows $V[k]$ only for the interval $0 \leq k \leq 63$. Which of the following signals could be $x[n]$? That is, which are consistent with the information shown in the figure?

$$x1[n] = \cos(\pi n/4) + \cos(0.26\pi n)$$

$$x2[n] = \cos(\pi n/4) + (1/3)\sin(\pi n/8)$$

$$x3[n] = \cos(\pi n/4) + (1/3)\cos(\pi n/8)$$

$$x4[n] = \cos(\pi n/8) + (1/3)\cos(\pi n/16)$$

$$x5[n] = (1/3)\cos(\pi n/4) + \cos(\pi n/8)$$

$$x6[n] = \cos(\pi n/4) + (1/3)\cos(\pi n/8 + \pi/3)$$

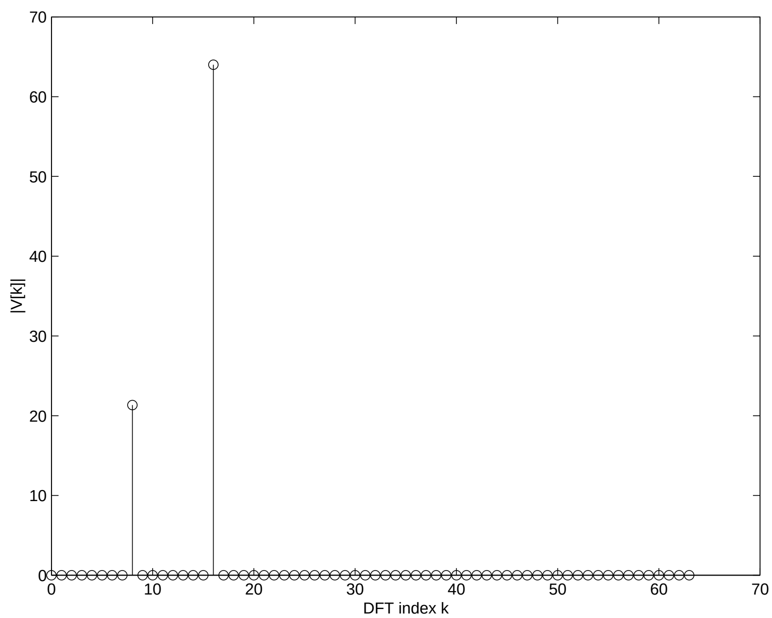

Figure 4.1: 128-point DFT