Problem 7.1
Figure 1 shows the DFS, \( \tilde{X}_1[k] \), of a periodic sequence \( \tilde{x}_1[n] \) that has period \( N = 4 \). Figure 2 shows the periodic sequence \( \tilde{x}_2[n] \) (period \( N = 4 \)).

(a) Determine the periodic sequence \( \tilde{x}_1[n] \).

(b) Find the sequence \( \tilde{y}_1[n] \) whose DFS is equal to the product of the DFS of \( \tilde{x}_1[n] \) and the DFS of \( \tilde{x}_2[n] \), i.e., \( \tilde{Y}_1[k] = \tilde{X}_1[k] \tilde{X}_2[k] \).

(c) Suppose that \( \tilde{x}_1[n] \) is the input to a filter with frequency response \( h[n] = \left( \frac{1}{2} \right)^n u[n] \). What is the output of the filter? (An analytical expression should be fairly easy to obtain.)
Problem 7.2
*Oppenheim and Schaefer*, problem 8.28

Problem 7.3
*Oppenheim and Schaefer*, problem 8.29

Problem 7.4
Compute the DFT of each of the following finite-length sequences considered to be of length $N$ (where $N$ is even):

(a) $x[n] = \delta[n]$

(b) $x[n] = \delta[n - n_0]$ \hspace{1cm} $0 \leq n_0 \leq N - 1$

(c) $x[n] = \begin{cases} 
1, & n \text{ even}, \hspace{0.5cm} 0 \leq n \leq N - 1 \\
0, & n \text{ odd}, \hspace{0.5cm} 0 \leq n \leq N - 1 
\end{cases}$

(d) $x[n] = \begin{cases} 
1, & 0 \leq n \leq N/2 - 1 \\
0, & N/2 \leq n \leq N - 1 
\end{cases}$

(e) $x[n] = \begin{cases} 
\alpha^n, & 0 \leq n \leq N - 1 \\
0, & \text{otherwise} 
\end{cases}$