ECE 535 DIGITAL SIGNAL PROCESSING

Problem Set 9

Spring 2001

Issued: Tuesday, April 10, 2001 **Due:** Tuesday, April 17, 2001

Reading in *Oppenheim and Schafer*

Week of 4/10/01	—	Chapter 7
		Section 5.7
		Appendix B

Although this problem set may seem long, note that problems 9.2, 9.3, and 9.4 require VERY short answers.

Problem 9.1

The decimation-in-time FFT algorithm was developed in Section 9.3 for radix 2, *i.e.*, $N = 2^\nu$. A similar approach leads to a radix-3 algorithm when $N = 3^\nu$. Draw a flow graph for a 9-point decimation-in-time algorithm using a 3x3 decomposition of the DFT.

Problem 9.2

Oppenheim and Schafer, problem 7.3, part a only

Problem 9.3 Suppose we design a discrete-time filter using the impulse invariance technique with a CT lowpass filter as a prototype. The prototype filter has a cutoff frequency of $\Omega_c = 2\pi(1000)$ rad/sec and the impulse invariance transformation uses $T = 0.2$ ms. Assume that aliasing effects are negligible. What is the cutoff frequency ω_c of the resulting discrete-time filter?

Problem 9.4

We wish to design a discrete-time lowpass filter using the bilinear transformation on a continuous-time ideal lowpass filter. Assume that the continuous-time prototype has a cutoff frequency $\Omega_c = 2\pi(2000)$ rad/sec and we choose the bilinear transformation parameter $T = 0.4$ ms. What is the cutoff frequency ω_c of the resulting discrete-time filter?

Problem 9.5

Design a single-pole lowpass discrete-time filter with a 3-dB bandwidth of 0.2π using the bilinear transformation applied to the analog filter

$$H_c(s) = \frac{\Omega_c}{s + \Omega_c}$$

where Ω_c is the 3-dB bandwidth of the analog filter.

(a) Design the filter, *i.e.* determine $H(z)$.

(b) Sketch a structure for realizing this filter.

Problem 9.6

Suppose that you have a causal continuous-time filter defined by the system function:

$$H_c(s) = \frac{s + 0.1}{(s + 0.1)^2 + 9}.$$

(a) Use impulse invariance to determine the system function, $H(z)$ of a discrete-time filter.

(b) Sketch the pole-zero plot and the frequency response magnitude of the resulting discrete-time filter.

Problem 9.7

Consider designing a discrete-time filter with the system function $H(z)$ from a continuous-time filter with rational system function $H_c(s)$ by the transformation

$$H(z) = H_c(s)\Big|_{s=(1-z^{-2})/(1+z^{-2})} \ . \qquad (9.1\text{-}1)$$

(i) What contour does the $j\Omega$ axis in the s-plane map to in the z-plane? Justify your answer.

(ii) Determine the mapping between Ω and ω defined by this transformation. Sketch Ω as a function of ω.

(iii) Suppose that the continuous-time filter is a stable lowpass filter with passband frequency response such that

$$1 - \delta_1 \leq |H_c(j\Omega)| \leq 1 + \delta_1 \qquad \text{for } |\Omega| \leq 1.$$

If the discrete-time system is obtained by the transformation in Equation 9.1-1 above, determine the values of ω in the interval $|\omega| \leq \pi$ for which

$$1 - \delta_1 \leq |H(e^{j\omega})| \leq 1 + \delta_1.$$

Does this transformation provide a reasonable way to design DT lowpass filters from analog lowpass filter prototypes?