Although this problem set may seem long, note that problems 9.2, 9.3, and 9.4 require VERY short answers.

**Problem 9.1**
The decimation-in-time FFT algorithm was developed in Section 9.3 for radix 2, i.e., $N = 2^n$. A similar approach leads to a radix-3 algorithm when $N = 3^n$. Draw a flow graph for a 9-point decimation-in-time algorithm using a 3x3 decomposition of the DFT.

**Problem 9.2**
*Oppenheim and Schafer*, problem 7.3, part a only

**Problem 9.3**
Suppose we design a discrete-time filter using the impulse invariance technique with a CT lowpass filter as a prototype. The prototype filter has a cutoff frequency of $\Omega_c = 2\pi(1000)$ rad/sec and the impulse invariance transformation uses $T = 0.2$ ms. Assume that aliasing effects are negligible. What is the cutoff frequency $\omega_c$ of the resulting discrete-time filter?

**Problem 9.4**
We wish to design a discrete-time lowpass filter using the bilinear transformation on a continuous-time ideal lowpass filter. Assume that the continuous-time prototype has a cutoff frequency $\Omega_c = 2\pi(2000)$ rad/sec and we choose the bilinear transformation parameter $T = 0.4$ ms. What is the cutoff frequency $\omega_c$ of the resulting discrete-time filter?

**Problem 9.5**
Design a single-pole lowpass discrete-time filter with a 3-dB bandwidth of $0.2\pi$ using the bilinear transformation applied to the analog filter

$$H_c(s) = \frac{\Omega_c}{s + \Omega_c}$$

where $\Omega_c$ is the 3-dB bandwidth of the analog filter.

(a) Design the filter, i.e. determine $H(z)$.

(b) Sketch a structure for realizing this filter.

**Problem 9.6**
Suppose that you have a causal continuous-time filter defined by the system function:

$$H_c(s) = \frac{s + 0.1}{(s + 0.1)^2 + 9}.$$ 

(a) Use impulse invariance to determine the system function, $H(z)$ of a discrete-time filter.
(b) Sketch the pole-zero plot and the frequency response magnitude of the resulting discrete-time filter.

**Problem 9.7**

Consider designing a discrete-time filter with the system function $H(z)$ from a continuous-time filter with rational system function $H_c(s)$ by the transformation

$$H(z) = H_c(s) \bigg|_{s = (1 - z^{-2})/(1 + z^{-2})}.$$  \hspace{1cm} (9.1-1)

(i) What contour does the $j\Omega$ axis in the $s$-plane map to in the $z$-plane? Justify your answer.

(ii) Determine the mapping between $\Omega$ and $\omega$ defined by this transformation. Sketch $\Omega$ as a function of $\omega$.

(iii) Suppose that the continuous-time filter is a stable lowpass filter with passband frequency response such that

$$1 - \delta_1 \leq |H_c(j\Omega)| \leq 1 + \delta_1 \quad \text{for} \quad |\Omega| \leq 1.$$  

If the discrete-time system is obtained by the transformation in Equation 9.1-1 above, determine the values of $\omega$ in the interval $|\omega| \leq \pi$ for which

$$1 - \delta_1 \leq |H(e^{j\omega})| \leq 1 + \delta_1.$$  

Does this transformation provide a reasonable way to design DT lowpass filters from analog lowpass filter prototypes?