A real continuous-time signal \( x_c(t) \) is bandlimited to frequencies below 10 kHz, i.e., \( X_c(j\Omega) = 0 \) for \( |\Omega| \geq 2\pi(10000) \). The signal \( x_c(t) \) is sampled with a sampling rate of 20 kHz to produce a sequence \( x[n] = x_c(nT) \) with \( T = 0.5 \times 10^{-4} \) seconds. The \( N \)-point DFT \( X[k] \) of \( N = 1000 \) samples of \( x[n] \) is computed.

(a) What continuous-time frequency does \( X[k = 150] \) correspond to?

(b) What continuous-time frequency does \( X[k = 800] \) correspond to?

Problem ECE535-6 (Old exam question worth 25 points)

(a) For part (a) of this problem \( x_1[n] \) refers to a 6-point sequence which has a real 6-point DFT \( X_1[k] \) as shown in Figure 1.

(i) Determine \( x_1[0] \).
(ii) Sketch and clearly label a plot of \( X_2[k] \), the 6-point DFT of \( x_2[n] \) where
\[
x_2[n] = (-1)^n x_1[n] \quad 0 \leq n \leq 5.
\]

(b) The 6-point sequence \( Y[k] \) is defined as
\[
Y[k] = X_4[k] X_5[k]
\]
where \( X_4[k] \) and \( X_5[k] \) are the 6-point DFT’s of the 6-point sequences \( x_4[n] \) and \( x_5[n] \), respectively. All that is known about these two sequences is that \( x_4[0] = 0 \). The sequence \( y[n] \) is the 6-point inverse DFT of \( Y[k] \). Let \( y_{\text{lin}}[n] \) be the result of the linear convolution of \( x_4[n] \) and \( x_5[n] \). Specify the set of values of \( y_{\text{lin}}[n] \) that can be obtained from \( y[n] \) and how you would extract them from \( y[n] \). Note that your method of extraction only has to work for the \( x_4 \) and \( x_5 \) sequences described above – not for general 6-point sequences.