Problem ECE535-9  In this problem you will use the window method to design an FIR linear-phase, causal discrete time filter that approximates the ideal response:

\[ H_d(e^{j\omega}) = \begin{cases} 
1 & |\omega| \leq \frac{\pi}{4} \\
0 & \frac{\pi}{4} < |\omega| \leq \pi 
\end{cases} \]

(a) Obtain an expression for the infinite duration impulse response \( h_d[n] \).

(b) Determine the coefficients of a 5-tap causal filter using a Hamming window.

(c) What is the group delay of this filter?

See additional problem on back of this page.
Problem ECE535-10 (Old exam question worth 25 points)
Let $s[n]$ be an $N$-point sequence, where $N$ is even, such that $s[n] = 0$ for $n < 0$ and $n > N - 1$. We would like to compute samples
\[ T[k] = S(e^{j\omega}) \bigg|_{\omega = \frac{2\pi k}{N/2}} \quad k = 0, \ldots, \frac{N}{2} - 1 \]
that is, $(N/2)$ samples in frequency, evenly-spaced between $\omega = 0$ and $\omega = 2\pi$. Five methods are proposed below for computing the desired samples, each yielding a different sequence $T_1[k], \ldots, T_5[k]$. For each method, state whether the computed samples $T_i[k]$ are guaranteed to be equal to the desired samples $T[k]$. Justify your answer.

Method 1: Define
\[ t_1[n] = \begin{cases} s[n], & 0 \leq n \leq (N/2) - 1 \\ 0, & \text{otherwise} \end{cases} \]
Let $T_1[k]$ be the $(N/2)$-point DFT of $t_1[n]$.

Method 2: Define
\[ t_2[n] = \begin{cases} s[n] + s[n + (N/2)], & 0 \leq n \leq (N/2) - 1 \\ 0, & \text{otherwise} \end{cases} \]
Let $T_2[k]$ be the $(N/2)$-point DFT of $t_2[n]$.

Method 3: Define
\[ t_3[n] = \begin{cases} s[2n] + s[2n + 1], & 0 \leq n \leq (N/2) - 1 \\ 0, & \text{otherwise} \end{cases} \]
Let $T_3[k]$ be the $(N/2)$-point DFT of $t_3[n]$.

Method 4: Define
\[ q_1[n] = \begin{cases} s[n], & 0 \leq n \leq (N/2) - 1 \\ 0, & \text{otherwise} \end{cases} \quad q_2[n] = \begin{cases} s[n + (N/2)], & 0 \leq n \leq (N/2) - 1 \\ 0, & \text{otherwise} \end{cases} \]
Let $Q_1[k]$ and $Q_2[k]$ be the $(N/2)$-point DFT’s of $q_1[n]$ and $q_2[n]$, respectively. Let $T_4[k] = Q_1[k] + Q_2[k]$.

Method 5: Define
\[ t_5[n] = \begin{cases} W_N^n (s[n] - s[n + (N/2)]), & 0 \leq n \leq (N/2) - 1 \\ 0, & \text{otherwise} \end{cases} \]
where $W_N = e^{-j(2\pi/N)}$. Let $T_5[k]$ be the $(N/2)$-point DFT of $t_5[n]$. 
