ECE 535 DIGITAL SIGNAL PROCESSING

Practice Problems

April 27, 2004

Problem 1

Figure P1 shows the magnitude $|V[k]|$ of the 128-point DFT $V[k]$ for a signal $v[n]$. The signal $v[n]$ was obtained by multiplying $x[n]$ by a 128-point rectangular window $w[n]$, i.e. $v[n] = x[n]w[n]$. Note that Figure P3-1 shows $V[k]$ only for the interval $0 \leq k \leq 63$. Which of the following signals could be $x[n]$? That is, which are consistent with the information shown in the figure?

$$x1[n] = \cos(\pi n/4) + \cos(0.26\pi n)$$

$$x2[n] = \cos(\pi n/4) + (1/3)\sin(\pi n/8)$$

$$x3[n] = \cos(\pi n/4) + (1/3)\cos(\pi n/8)$$

$$x4[n] = \cos(\pi n/8) + (1/3)\cos(\pi n/16)$$

$$x5[n] = (1/3)\cos(\pi n/4) + \cos(\pi n/8)$$

$$x6[n] = \cos(\pi n/4) + (1/3)\cos(\pi n/8 + \pi/3)$$

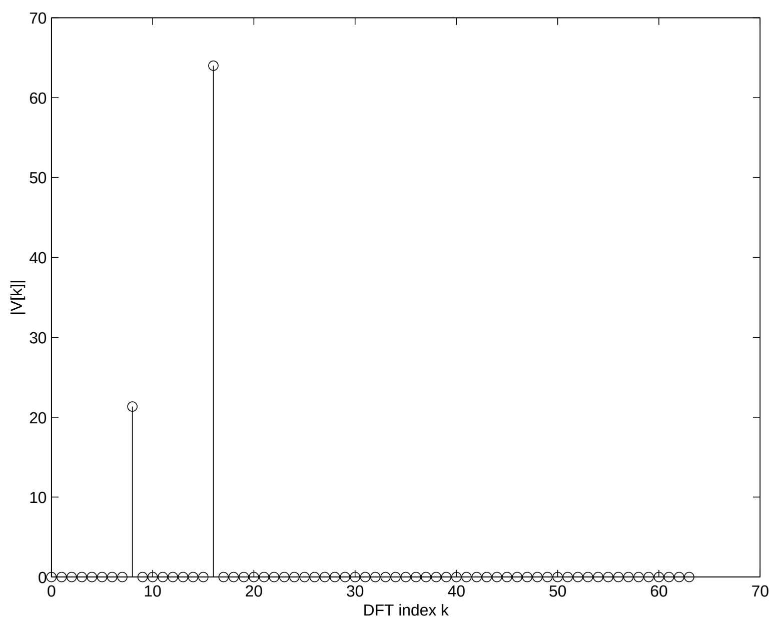

Figure P1

Problem 2 (10 points)
The system shown in Figure 2.1 is a spectral analyzer for the continuous-time signal $x_c(t)$. The signal is sampled at a rate of 50 Hz and the resulting discrete-time signal $x[n]$ is windowed with a 201-point Hamming window. The system takes a 1000-point DFT of the windowed signal and stores the result in $V[k]$ for $k = 0, \ldots, 999$.

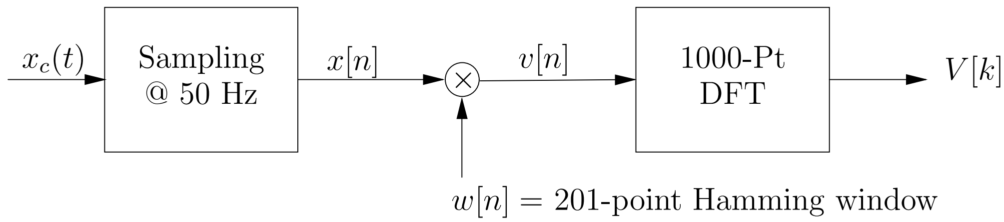

Figure 2.1:

Answer the following questions about the system. If you need information about common window properties, see page 2.

1. What is the equivalent frequency spacing in continuous-time of adjacent frequency samples in $V[k]$? In other words what is the spacing in Hz between $V[0]$ and $V[1]$?
2. To what continuous-time frequency does the index $k = 800$ in $V[k]$ correspond?
3. Suppose that $x_c(t)$ consists of two sinusoids: $x_c(t) = \cos(\Omega_0 t) + \cos((\Omega_0 + \Delta\Omega)t)$. What is the smallest value of $\Delta\Omega$ for which a plot of $|V[k]|$ will show two distinct spectral peaks? Justify your answer (indicating what, if any, approximations you have made).

Comparison of commonly used windows (from Oppenheim/Schafer/Buck, Table 7.1). Note that the window length is $M + 1$.

Type of Window	Peak Sidelobe Amplitude (dB relative to peak)	Approximate Width of Mainlobe	Peak Approximation Error $20 \log_{10} \delta$ (dB)
Rectangular	-13	$\frac{4\pi}{M+1}$	-21
Bartlett	-25	$\frac{8\pi}{M}$	-25
Hanning	-31	$\frac{8\pi}{M}$	-44
Hamming	-41	$\frac{8\pi}{M}$	-53
Blackman	-57	$\frac{12\pi}{M}$	-74

Problem 3 (10 points)

This problem concerns the spectral analysis of a 128-point signal $x[n]$. This signal is windowed with a 128-point rectangular window to get the DTFT $|X_{rect}(e^{j\omega})|$ and with a 128-point Hamming window to obtain the DTFT $|X_{ham}(e^{j\omega})|$. Figure 3.1 shows both of these magnitude spectra.

Listed below are five different possible signals $x_1[n]$ through $x_5[n]$. Indicate for each of these signals if it could be the signal $x[n]$ which produced $|X_{rect}(e^{j\omega})|$ and $|X_{ham}(e^{j\omega})|$. Note that it is possible that more than one or none of the signals listed could produce the graphs given. You may find Table 3.1 useful. **Justify your answer clearly. Answers without justification will be assumed to be guesses and receive no credit.**

$$\begin{aligned}
x_1[n] &= \sin(0.28\pi n) + 0.012\sin(0.45\pi n + \pi/2) \\
x_2[n] &= 0.5\sin(0.28\pi n) + 0.5\sin(0.281\pi n) + 0.012\sin(0.45\pi n + \pi/2) \\
x_3[n] &= \sin(0.28\pi n) + 0.001\sin(0.45\pi n + \pi/2) \\
x_4[n] &= \sin(0.28\pi n) + 0.012\sin(0.45\pi n) \\
x_5[n] &= 0.5\sin(0.28\pi n) + 0.5\sin(0.31\pi n) + 0.012\sin(0.45\pi n + \pi/2)
\end{aligned}$$

Window Type of Length $M+1$	Approximate Width of Mainlobe	Peak Side-Lobe Amplitude (dB)
Rectangular	$4\pi/(M+1)$	-13
Hamming	$8\pi/M$	-41

Table 3.1: Commonly Used Windows

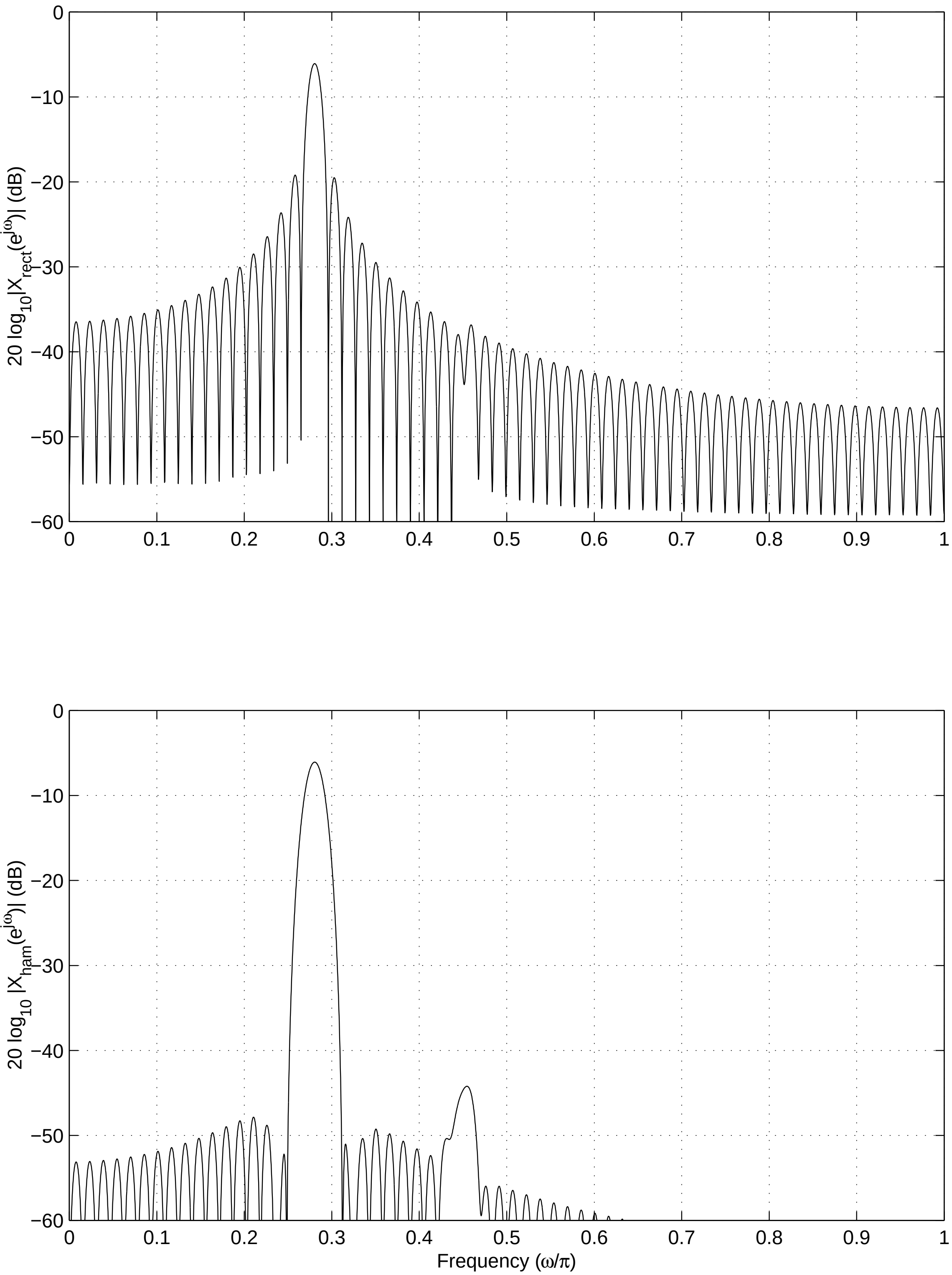

Figure 3.1: DTFT Magnitudes $|X_{rect}(e^{j\omega})|$ and $|X_{ham}(e^{j\omega})|$