Consider the LFM signal that is received on the array of sensors:

\[ s_{\text{src}}(t) = A \cos \left( \Delta_0 t + \frac{\Pi W t^2}{T} \right) \]

Source sends out signal \( s_{\text{src}}(t) \)

Signal received on sensor \( \phi \) is a delayed version of \( s_{\text{src}}(t) \)

\[ s_{\phi}(t) = B \cos \left( \Delta_0 (t-t_0) + \frac{\Pi W (t-t_0)^2}{T} \right) \]

\( B \): A signal time delay of 60 seconds

(It is time signal takes to propagate from source to receiver sensor \( \phi \))

In general, we don't know \( t_0 \).

**Note:** Using Euler's we can rewrite \( s_{\phi}(t) \):

\[ s_{\phi}(t) = \frac{B}{2} \left( e^{-j(\Delta_0 (t-t_0) + \frac{\Pi W (t-t_0)^2}{T})} - e^{-j(2\Delta_0 t - \Delta_0 t_0 + \frac{\Pi W t^2}{T})} \right) \]

We demodulate by multiplying by \( e^{-j\Delta_0 t} \):

\[ s_{\phi \text{ demod}}(t) = \frac{B}{2} \left( e^{-j(\Delta_0 t_0 + \frac{\Pi W (t-t_0)^2}{T})} - e^{-j(2\Delta_0 t - \Delta_0 t_0 + \frac{\Pi W t^2}{T})} \right) \]

Baseband

Two times freq
AFTER IDEAL LOWPASS FILTERING, DOUBLE FREQ. TERM IS GONE

\[ S_{\text{LPF}}(t) = \frac{B}{2} e^{-\frac{j \pi W (t-t_0)^2}{T}} \]

COMPLEX SCALAR.

WHEN WE MATCHED FILTER, WE CONVOLVE WITH

\[ S_{\text{MF}}(t) = (e^{j \frac{\pi W (-t)^2}{T}})^* \quad -\frac{T}{2} \leq t \leq \frac{T}{2} \]

\[ = e^{-j \frac{\pi W (t^2)}{T}} \]

(THE MF IS THE TIME-REVISED AND CONJUGATED VERSION OF THE ORIGINAL BASEBAND SIGNAL).

\[ S_{\text{LPF}}(t) \ast S_{\text{MF}}(t) \text{ YIELDS A PEAK WHEN } t = t_0 \]

[ASSUMING WE'VE IMPLEMENTED NON-CAUSAL FILTERING]

TO UNDERSTAND WHAT PHASE ADJUSTMENTS MIGHT BE NECESSARY FOR BEAMFORMING CONSIDER HOW SIGNAL RECEIVED ON SENSOR 1 DIFFERS FROM THE ONE RECEIVED ON SENSOR 0,

NOTE: \[ t_1 = t_0 + \Delta t \]

DIFFERENCE IN ARRIVAL TIME BETWEEN SENSOR 0 AND SENSOR 1.