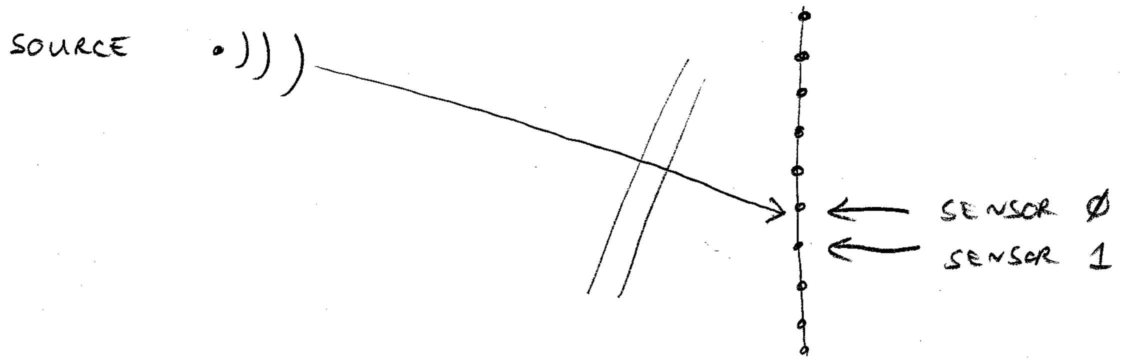
CONSIDER THE LFM SIGNAL THAT IS RECEIVED ON THE ARRAY OF SENSORS:



SOURCE SENDS OUT SIGNAL $s_{SRC}(t)$

$$s_{SRC}(t) = A\cos\left(\Omega_0 t + \frac{\pi W t^2}{T}\right)$$

SIGNAL RECEIVED ON SENSOR Ø IS A DELAYED VERSION OF $s_{SRC}(t)$

$$s_{\phi}(t) = B\cos\left(\Omega_0 (t-t_0) + \frac{\pi W (t-t_0)^2}{T}\right)$$

$B = A \cdot$ SIGNAL LOSS

TIME DELAY OF t_0 SECONDS

(IT IS TIME SIGNAL TAKES TO PROPAGATE FROM SOURCE TO RECEIVER SENSOR Ø)

IN GENERAL WE DON'T KNOW t_0.

<u>NOTE</u>: USING EULER'S WE CAN REWRITE $s_{\phi}(t)$:

$$s_{\phi}(t) = \frac{B}{2}\left(e^{j\left(\Omega_0(t-t_0) + \frac{\pi W (t-t_0)^2}{T}\right)} + e^{-j\left(\Omega_0(t-t_0) + \frac{\pi W (t-t_0)^2}{T}\right)} \right)$$

WE DEMODULATE BY MULTIPLYING BY $e^{-j\Omega_0 t}$:

$$s_{\phi\,DEMOD}(t) = \frac{B}{2}\left(\underbrace{e^{j\left(-\Omega_0 t_0 + \frac{\pi W (t-t_0)^2}{T}\right)}}_{\text{BASEBAND}} + \underbrace{e^{-j\left(2\Omega_0 t - \Omega_0 t_0 + \frac{\pi W (t-t_0)^2}{T}\right)}}_{\text{TWO TIMES FREQ}} \right)$$

AFTER IDEAL LOWPASS FILTERING, DOUBLE FREQ. TERM IS GONE

$$S_{\phi LPF}(t) = \frac{B}{2} \underbrace{e^{-j\Omega_0 t_0}}_{\text{COMPLEX SCALAR.}} e^{j\frac{\pi W (t-t_0)^2}{T}}$$

WHEN WE MATCHED FILTER, WE CONVOLVE WITH

$$S_{MF}(t) = \left(e^{j\frac{\pi W(-t)^2}{T}}\right)^* \quad \frac{-T}{2} \le t \le \frac{T}{2}$$

$$= e^{-j\frac{\pi W(t^2)}{T}}$$

(THE MF IS THE TIME-REVERSED AND CONJUGATED VERSION OF THE ORIGINAL BASEBAND SIGNAL).

$S_{\phi LPF}(t) * S_{MF}(t)$ YIELDS A PEAK WHEN $t = t_0$

[ASSUMING WE'VE IMPLEMENTED NON-CAUSAL FILTERING]

TO UNDERSTAND WHAT PHASE ADJUSTMENTS MIGHT BE NECESSARY FOR BEAMFORMING CONSIDER HOW SIGNAL RECEIVED ON SENSOR 1 DIFFERS FROM THE ONE RECEIVED ON SENSOR Ø.

NOTE: $t_1 = t_0 + \underbrace{\Delta t_1}_{\text{DIFFERENCE IN ARRIVAL TIME BETWEEN SENSOR Ø AND SENSOR 1.}}$