ECE-738 Problem 10

Consider the complex Gaussian random variable \( \tilde{a} \):

\[
\tilde{a} = a_R + j a_I
\]

where \( a_R \) and \( a_I \) are real independent Gaussian random variables with zero mean and variance equal to 1.

(a) Determine the mean and variance of the complex variable \( \tilde{a} \).

(b) Suppose you want to generate random realizations of a zero-mean complex Gaussian variable \( \tilde{b} \) with variance equal to \( \sigma_b^2 \). 1000 random realizations of this variable can be generated with the following Matlab commands:

\[
\begin{align*}
N &= 1000; \\
\text{btilde} &= C \cdot \text{randn}(1,N) + j \cdot D \cdot \text{randn}(1,N) + E;
\end{align*}
\]

where \( C, D, \) and \( E \) are constants. Determine the values of \( C, D, \) and \( E \) which will ensure that the samples of \( \tilde{b} \) have zero mean and variance \( \sigma_b^2 \).

Note: We often model a planewave signal incident on an array as follows:

\[
x = \tilde{b} \sqrt{M} v(\phi) + w,
\]

where \( v(\phi) \) is the array response vector and \( \tilde{b} \) is a zero-mean complex Gaussian scalar random variable with variance \( \sigma_b^2 \). The \( w \) is a vector containing zero-mean complex Gaussian noise. In the examples we considered in class last Monday night, the noise vector was spatially white, hence \( R_w = \sigma_w^2 \text{I} \). To generate multiple realizations of \( x \), we assume that the \( \tilde{b} \) for each realization is uncorrelated from one realization to the next and that the noise is also uncorrelated from one realization to the next. Thus, we would generate \( N \) independent samples of \( \tilde{b} \) and multiply them by the scaled array response vector and then add noise. For example, following the Matlab commands will generate snapshots of a planewave signal incident on a uniform line array with angle \( \phi \). The array is assumed to have spacing \( d \) and the signal has wavelength \( \lambda \).

\[
\begin{align*}
u0 &= d \cdot \text{sin}(\phi) / \text{lambda}; \\
v &= 1 / \sqrt{M} \cdot \text{exp}(-j \cdot 2 \cdot \pi \cdot (0:M-1) \cdot v0); \\
x &= \sqrt{M} \cdot v \cdot \text{btilde};
\end{align*}
\]

Note that the \( \text{btilde} \) vector is defined as it was in Problem 10 above. Also, note that for Matlab, \( \text{phi} \) is assumed to be in radians, not degrees. This code generates \( N \) snapshots received by an \( M \) element array. In general you would add noise to these snapshots as well. The above discussion provides a general idea of how to simulate the data required for Problem 11.4 below.

Problem 11.4 in Manolakis, Ingle, Kogon