ECE 738 Advanced Digital Signal Processing

Problem Set 7

Spring 2006

Issued: Thursday, May 4, 2006 **Due:** For practice only; not due.

Reading in *Manolakis, Ingle, and Kogon*

4/3/06	—	Section 11.0-11.2
4/10/06	—	Section 11.3
4/24/06	—	Section 11.4
5/1/06	—	Section 11.5

The following problems review essential aspects of the beamforming problem for linear arrays. While many of the results were derived in class or in the textbook, I encourage you to spend some time working through these problems on your own.

ECE-738 Problem 11

Consider a uniform line array with M sensors. The spacing between the sensors is d meters. As in the Manolakis/Ingle/Kogon textbook, we define ϕ to be the angle of arrival, measured from broadside. The variable u is the normalized spatial frequency,

$$u = \frac{d \sin(\phi)}{\lambda},$$

where λ is the wavelength of the signal. The beam response is defined as a function of u or ϕ:

$$C(u) = \mathbf{c}^H \mathbf{v}(u) \qquad \text{or} \qquad C(\phi) = \mathbf{c}^H \mathbf{v}(\phi),$$

where $\mathbf{c}$ is the weight vector and $\mathbf{v}(u)$ is the array response vector corresponding to a particular spatial frequency (or angle). The beampattern is defined as $|C(u)|^2$ or $|C(\phi)|^2$.

(a) Determine an analytical expression for the beampattern of the spatial matched filter steered towards angle $\phi = \phi_s$. First, write the beampattern as a function of the normalized frequency u. Then write the expression as a function of angle ϕ.

(b) Using the results of part a, determine an expression for the null-to-null beamwidth $\Delta\phi_{nn}$. Note that $\frac{1}{2}\Delta\phi_{nn}$ is called the *Rayleigh resolution limit*. Plane waves arriving at angles separated by more than $\frac{1}{2}\Delta\phi_{nn}$ should be resolvable by the array.

(c) Evaluate $\Delta\phi_{nn}$ for an array steered to broadside ($\phi = 0$) and to endfire ($\phi = \frac{\pi}{2}$). How do the beamwidths (and hence the resolving capabilities) compare?

(d)
  (i) Consider the beam response as a function of u. $C(u)$ is a periodic function that is defined for $-\infty < u < +\infty$. What is the period of $C(u)$?

  (ii) Propagating signals have angles that lie between $-\frac{\pi}{2} \leq \phi \leq +\frac{\pi}{2}$. Thus, while $C(u)$ is defined for $-\infty < u < +\infty$, it only represents propagating signals when u corresponds to an angle between $-\frac{\pi}{2}$ and $+\frac{\pi}{2}$. This is known as the *visible region*. What values of u correspond to the visible region?

  (iii) The periodic replications of the mainlobe in C are called *grating lobes*. If grating lobes lie within the visible region, there is ambiguity about which direction a signal is coming from, *i.e.*, spatial aliasing. If we want to be able to steer the array to angles between $-\frac{\pi}{2}$ and $+\frac{\pi}{2}$, what sensor spacing is sufficient to keep the grating lobes out of the visible region?

Problem 11.10 in *Manolakis, Ingle, Kogon*

Problem 11.14 in *Manolakis, Ingle, Kogon*