Interpolation mehods for vertical linear array element localization

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September 2008

Proceedings of the 2008 IEEE/MTS Oceans Conference

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***Abstract*— Array element localization is crucial for applications such as ocean acoustic tomography. Loss of navigation data makes it difficult to compensate for array motion when implementing operations such as mode filtering or beamforming. This paper presents a method for estimating missing array navigation data using an Empirical Orthogonal Function (EOF) model. The method can be applied to estimate the location of some vertical array elements based on the location of the other elements. It assumes that second order statistics can be estimated from a set of navigation measurements for the full array. The paper applies the EOF-based method to estimate missing navigation data for the Long range Ocean Acoustic Propagation EXperiment (LOAPEX). The results are evaluated by examining how the errors in mooring motion estimation affect mode processing. In particular the paper analyzes the degradation in array gain and the errors in time of arrival for the low order modes. The error statistics indicate that use of the EOF method has a negligible effect on mode processing.**

I. Introduction

Array element localization is crucial for applications such as ocean acoustic tomography. Spatial filtering for modes relies on a coherent phase across the entire array. Lack of phase compensation for array motion in mode filtering causes loss in spatial gain in addition to time of arrival errors for the modes. Tomography experiments usually track the array elements by means of acoustic transponders deployed near the array. Navigation dropouts make it difficult to compensate for array motion. The Long Range Ocean Acoustic Propagation Experiment (LOAPEX) [1] had some navigation data missing. This paper discusses the estimation of navigation data for LOAPEX.

LOAPEX was conducted in the fall of 2004 in conjunction with the SPICE04 [2] experiment. Figure 1 shows a diagram of the Vertical Line Array (VLA) used to measure the low mode signals during LOAPEX and SPICE04. The VLA consisted of 40 hydrophones spanning depths of 350 m to 1750 m. The VLA had two AVATOC recorders for the top and the bottom halves of the VLA. The AVATOC disks recorded the acoustic data for each set of 20 hydrophones. A transponder network tracked the hydrophones and wrote the navigation data for the top and the bottom halves of the array to the respective AVATOCs. For more details on the acoustic tracking see the paper [3]. Unfortunately, due to disk failures and other technical issues, the navigation data was occasionally lost for half the VLA. Since lack of array element localization causes time of arrival errors and loss of spatial gain, the missing navigation data must be estimated in order to use the signals at the VLA.

Estimation of the navigation data for one half of the VLA from the other half is an extrapolation problem across depth. The extrapolation method suitable for this problem should take into account the general shape of the array. This paper uses a basis defined by the Empirical Orthogonal Functions (EOFs) to describe the shape of the array. Note that EOF analysis is the same as Principal Component Analysis (PCA) [4] or Karhunen-Loeve expansion [5] of the data. See the book by Jollife [4] for an overview on the PCA method and its applications. Reference [6] discusses estimation of missing data in images via PCA.

The paper is organized as follows. Section II presents the EOF approach and shows how the coefficients for the expansion can be computed via Least Squares (LS). Section III discusses the application of the EOF method to the LOAPEX data set and shows the statistics of the estimation error. Section IV investigates the sensitivity of mode processing to mooring estimation errors. Specifically, this section shows how array navigation errors affect the spatial gain of the matched mode filter and mode time-of-arrival calculations. The statistics show that the estimation algorithms for array element localization work well with only a minor degradation in mode processing performance.

II. EOF based reconstruction

This section describes a method of estimating missing navigation data using a set of EOFs derived from the second order statistics of the navigation data. The approach assumes that these statistics can be estimated during times when complete navigation information is available. The EOFs are the eigenvectors of the correlation matrix, which form a complete orthonormal basis, thus the navigation data can be written as a weighted sum of the EOFs. When partial navigation information is available, *e.g.*, for half the array, the available data can be used to find a least squares estimate of the weights. This approach is convenient because it is straightforward to compute the EOFs from the data. It has the added advantage that it is possible to denoise the existing data by only using a subset of the EOFs for the reconstruction. The rest of this section describes the details of the approach and shows how the EOF coefficients are solutions to an LS optimization problem.

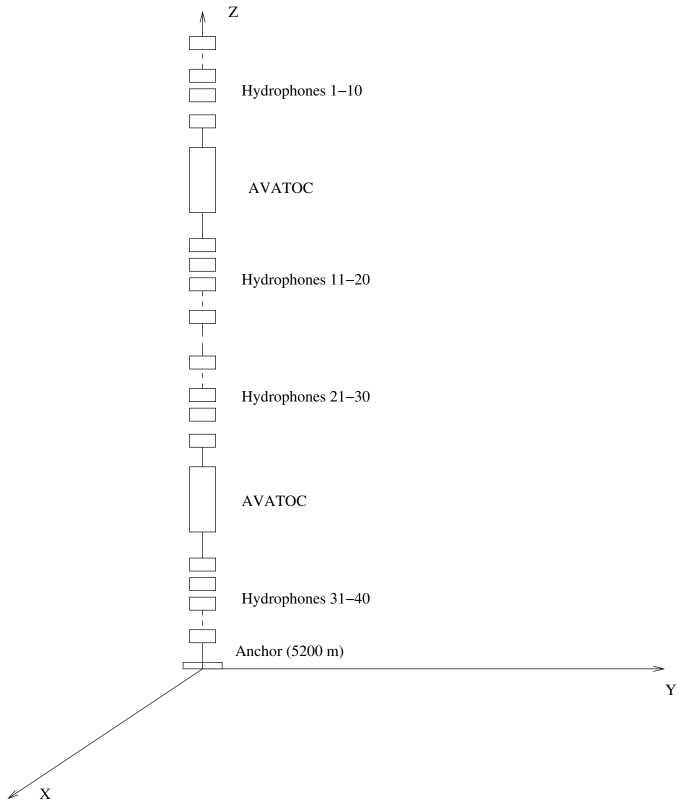


Fig. 1. The mooring of the LOAPEX VLA in a 3 dimensional coordinate system. The bottom and top halves of the VLA consisted of 20 hydrophones each. An acoustic system tracked the hydrophones and the navigation data of each half written to the corresponding AVATOCs. The **X**, **Y** and **Z** locations of the hydrophones were estimated after recovery.

Figure 1 shows a diagram of the construction of the array. The observation vector consists of the navigation data for the array arranged as a column such that

$$\mathbf{d} = \begin{bmatrix} \mathbf{X} \\ \cdots \\ \mathbf{Y} \\ \cdots \\ \mathbf{Z} \end{bmatrix}. \tag{1}$$

The vector $\mathbf{d}$ contains the navigation data in the X, Y and Z directions. The observation vector $\mathbf{d}$ is spanned a matrix of orthogonal column vectors U where,

$$\mathbf{d} = \mathbf{Uc}. \tag{2}$$

The coefficient vector $\mathbf{c}$ in Equation 2 is modeled as random and unknown. The columns of $\mathbf{U}$ constitute a complete orthonormal basis set and are the eigenvectors of the correlation matrix $\mathbf{R_{dd}}$. As mentioned earlier, it is assumed that for this problem, the eigenvectors $\mathbf{U}$ can be estimated from the data and hence shall be referred to as EOFs. Multiplying the previous equation by matrix L to accommodate for the missing data, Equation 2 now becomes,

$$\mathbf{Ld} = \mathbf{LUc}, \tag{3}$$

where $\mathbf{L}$ is a diagonal matrix that contains ones along the main diagonal except where the data is missing when the elements are set to zero. For instance the matrix $\mathbf{L}$ for the missing data in LOAPEX where half the array is missing is given by,

$$\text{diag}(\mathbf{L}) = \begin{bmatrix} \mathbf{0}\text{s (20x1 vector)} \\ \mathbf{1}\text{s (20x1 vector)} \\ \mathbf{0}\text{s (20x1 vector)} \\ \mathbf{1}\text{s (20x1 vector)} \\ \mathbf{0}\text{s (20x1 vector)} \\ \mathbf{1}\text{s (20x1 vector)} \end{bmatrix}. \tag{4}$$

The objective is to estimate the coefficient vector $\mathbf{c}$ and estimate the navigation data $\hat{d}$ by via the equation $\mathbf{d} = \mathbf{Uc}$. The estimate $\hat{\mathbf{c}}$ is obtained by minimizing the following function,

$$\text{J} = \underset{\text{c}}{\text{Min}} ||\mathbf{L}(\mathbf{d} - \mathbf{U}\hat{\mathbf{c}})||^{\mathbf{2}}. \tag{5}$$

The above function J is the square of the norm of the distance between the measured navigation data ($\mathbf{d}$) and the reconstructed navigation data ($\mathbf{d} = \mathbf{Uc}$). Minimizing J is a problem in LS filtering [7]. The LS estimate $\hat{\mathbf{c}}$ and the estimate $\mathbf{d}$ that contains estimates for the missing data are given by,

$$\hat{\mathbf{c}} = (\mathbf{U^T L U})^{-\mathbf{1}} \mathbf{U^T L d}, \tag{6}$$

$$\hat{\mathbf{d}} = \mathbf{U}(\mathbf{U^T L U})^{-\mathbf{1}} \mathbf{U^T L d}. \tag{7}$$

The next section explains the application of the EOF estimation scheme explained in this section to the LOAPEX data and also shows the results.

III. Results

During the yearlong deployment of SPICE04/LOAPEX, the VLA recorded navigation data once per hour. Some navigation data was lost lost during LOAPEX. It is reasonable to assume that the array dynamics are the same irrespective of the time of the experiment. The navigation data recorded during the overall SPICE04 experiment can be used to estimate the EOFs. The EOF estimator was calculated using the navigation samples recorded between yeardays 160 and 240, which correspond to the times before LOAPEX. The sample correlation matrix $\mathbf{R_{dd}}$ was constructed from the measured navigation data. The eigenvectors of $\mathbf{R_{dd}}$ were calculated to obtain the EOFs $\mathbf{U}$ of the data. Figure 2 shows a plot of the 10 most significant eigenvalues λ of the sample correlation matrix. The first 3 eigenvalues significantly differ in amplitude from the rest of the eigenvalues. Figure 2 also shows a plot of the measure μ_λ defined below.

$$\mu_\lambda = 100 \frac{\sum_{i=1}^{i=k} \lambda_i}{\sum_{i=1}^{i=N} \lambda_i}, k \le N, N = 120. \tag{8}$$

The measure μ_λ is equivalent to the percentage of energy contained in the subspace $\mathbf{U}$ for different number of eigenvectors. Figure 2 shows that the first 3 eigenvectors contain more than 99.99% of the total energy in the navigation data.

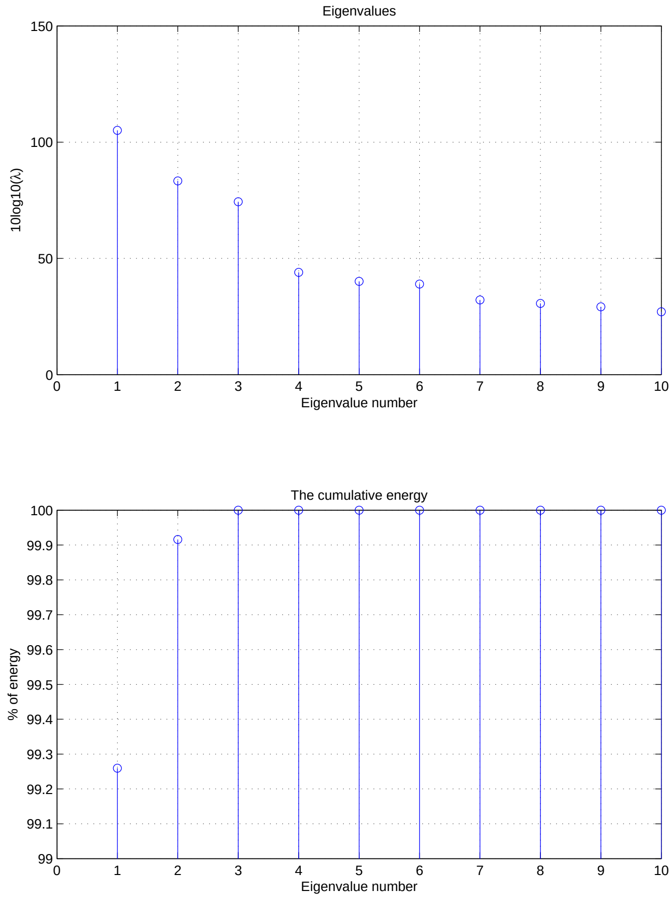


Fig. 2. The eigenvalues (λ) of the correlation matrix of the navigation data. The top subplot shows that the first 3 eigenvalues are at least 25 dB higher in amplitude than the rest of the eigenvalues. The bottom subplot shows a plot of the percentage of the cumulative energy contained in the eigenvalues.The first 3 eigenvalues make up more than 99.99 % of the energy in the navigation data.

Figure 3 shows a plot of the corresponding 3 EOFs. The first eigenvector mainly describes the depths ($\mathbf{Z}$), while the other two EOFs roughly describe the shape of the array in the $\mathbf{X}$ and $\mathbf{Y}$ directions. A reduced dimension EOF matrix $\hat{\mathbf{U}}$ consisting of only the three EOFs shown in Figure 3 was used to construct the EOF based LS estimator in Equation 6. As mentioned earlier in Section II, this has the advantage of denoising the estimates. Figure 4 shows a sample reconstruction of the navigation data for yearday 402. The EOF estimator estimated the top 20 hydrophones of the array from the other half. The figure shows the magnitude of the radial displacement of the array and the EOF reconstructed array, both in the direction of arrival of the signal. The reconstructed array shape closely follows the actual array shape.

The error statistics of the EOF-based LS filter were evaluated as follows. SPICE04 also had good navigation data for both the halves of the array for a contiguous time period between yeardays 400 to 480. Note that these yeardays correspond to times after LOAPEX. The EOFs shown in Figure 3 were used to estimate the navigation data of the upper half

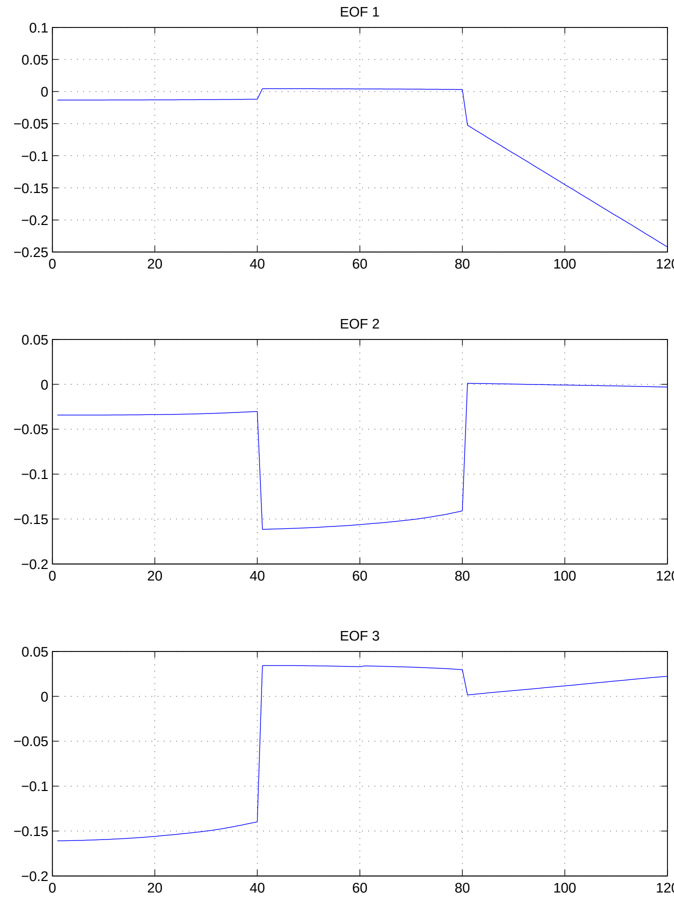


Fig. 3. EOFs 1 to 3 for the navigation data from yeardays 160 to 240. The 3 EOFs roughly describe the shape of the array in the X, Y and Z directions.

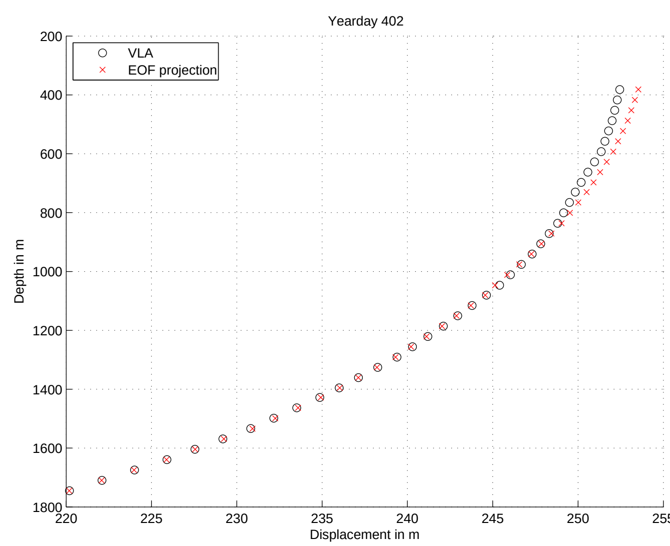


Fig. 4. EOF sample reconstruction for yearday 402. The black (○) curve shows the true position of the array and the red (×) curve shows the reconstructed array.

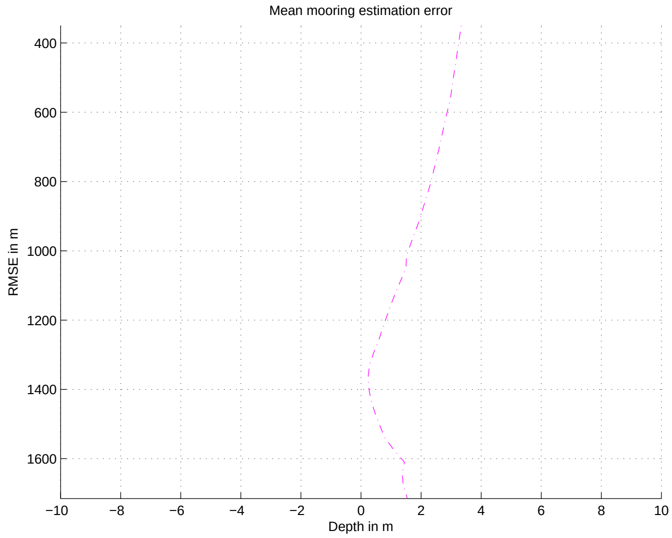


Fig. 5. The Root Mean Square Error (RMSE) performance of the EOF based reconstruction for yeardays 400 to 480 as a function of depth. The 3 EOFs shown in figure 3 were used in the EOF estimator. The RMSE decreases as a function of depth. The mean RMSE is in the order of 1.5 m.

from the lower half. The estimated navigation data was then compared with the actual navigation data. Figure 5 shows a plot of the mean error in the displacement of the array as a function of depth. The estimation error decreases with depth. This is due to the reason that the array moves less as depth increases. The shallowest hydrophone is off by a distance of approximately 3 m. The RMSE averaged across the entire array was around 1.5 m. The next section discusses the effect of the mooring estimation errors on mode processing.

IV. Sensitivity Analysis

Mode tomography relies on an accurate measure of time of arrival and spatial gain acquired via beamforming. Mooring estimation errors can cause both timing errors and loss of spatial gain. When the entire array suffers a mean displacement, the result is a time shift. Mooring errors that vary as a function of depth result in loss of phase coherence across the array and also cause mismatch in modeshapes. Both these effects result in a loss of spatial gain for mode beamforming.

Figure 5 shows that the mean mooring estimation error across the entire array is in the order of 1.5 m. The sound speed of the underwater channel is typically in the order of 1500 m/s, thus a mooring estimation error on the order of 1.5 m would thus result in timing errors of approximately 1 ms.

The narrowband model for the pressure field $\mathbf{P}$ is represented as,

$$\mathbf{P} = \sum_m a_m \mathbf{\Psi_m}, \tag{9}$$

where $\mathbf{\Psi_m}$ and a_m are the modeshape and mode amplitude for mode m respectively. Mode processing typically consists of a spatial filter or a beamformer that combines the pressure field $\mathbf{P}$ to measure the mode amplitude a_m. The Matched Filter (MF) beamformer is the most basic type mode filter. For an explanation of the MF beamformer see references by Ferris [8], Ingenito [9], or Wage [10]. The MF beamformer offers the maximum white noise gain for mode beamforming. The operation of the MF beamforming operation for mode m can be written as,

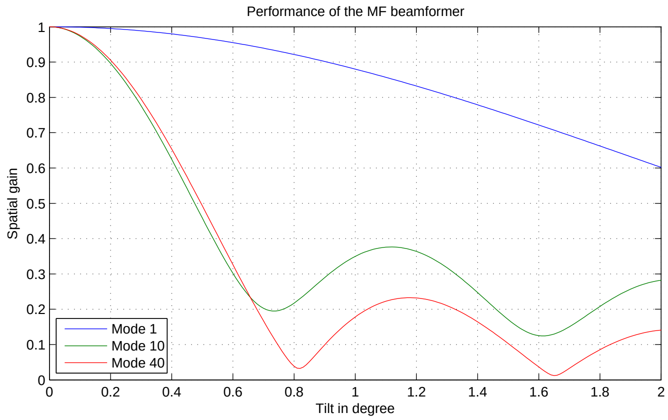
$$\hat{a_m} = \mathbf{\Psi_m^H} \mathbf{P}. \tag{10}$$

In the absence of crosstalk [10] from the high order modes, the output of the mode beamformer for mode m should be exactly equal to a_m. However, mooring estimation errors could result in a depth varying phase for the mode signal and also mismatch in the modeshape sampled at the array, both of which result in a loss of spatial gain for the mode beamformer. A simulation for the MF beamformer for modes 1 to 40 was performed for different values of array tilt. It was assumed that the input mode amplitudes a_m are equal to 1. Figure 6 shows a plot of the output ($\hat{a}_m$) of the Matched Filter (MF) beamformer with respect to tilt. Note that the output of the matched filter for input mode amplitude of 1 can also be interpreted as the spatial gain of the filter. The top subplot shows the spatial gain of the MF beamformer for different tilt angles. The tilt was measured with respect to the deepest hydrophone. For no array tilt, the output of the beamformer equals 1. For increasing values of tilt, Figure 6 shows loss in the amplitude of the modes at the output of the beamformer. As the modenumber increases, the modes vary more as a function of depth. Thus an increase in modenumber implies a greater sensitivity to mismatch. The higher order modes are thus more affected by tilt and the performance of the MF beamformer degrades faster for these modes. The tilt can also be measured in terms of the displacement of the shallowest hydrophone relative to the deepest hydrophone. The bottom subplot shows the performance of the MF beamformer with respect to the displacement of the shallowest hydrophone. The plot shows that the higher order modes suffer a 3 dB loss (0.5 decrease in amplitude) for displacements of approximately 10 m.

The mean estimation error as a function of depth in Figure 5 can also be interpreted as the mean tilt of the array. The plot in Figure 5 shows that the error in EOF estimation scheme could result in a maximum tilt of approximately 3 m for the shallowest hydrophone. Figure 7 shows a log plot of the spatial gain of the MF beamformer for a tilt of 3 m (with respect to the deepest hydrophone). The modes 1 to 40 do not suffer a loss of greater than 0.2 dB.

V. Summary

This paper presented an estimator for navigation data for missing navigation data. The estimator used an LS filter based on the EOFs of the navigation data. Section III applied the estimator to interpolate missing navigation data for LOAPEX. The reconstruction result shown in Section III demonstrated that the EOF estimator works well. The RMSE plotted as a function of depth showed that the EOF estimator has a mean



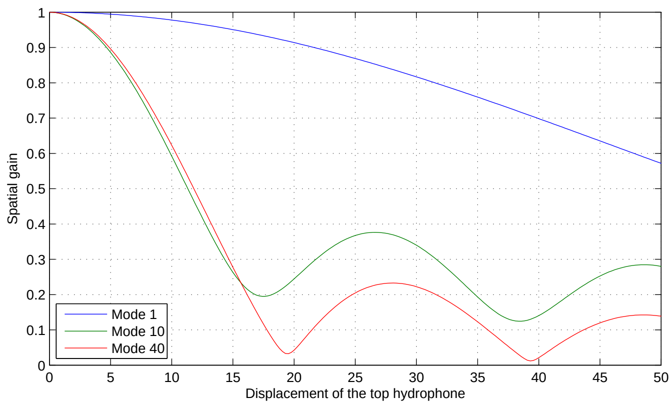


Fig. 6. The spatial gain of the MF beamformer for modes 1, 10 and 40 for different values of tilt. The tilt is measured with reference to the lowest hydrophone. Modes 10 and 40 are more sensitive to tilt than mode 1.

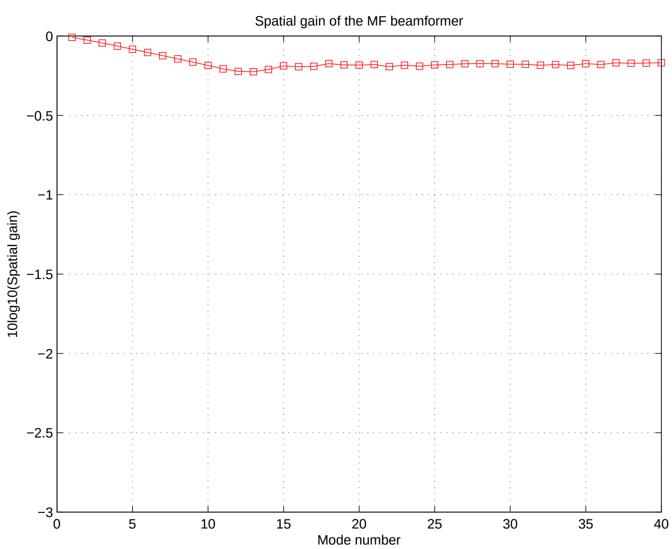


Fig. 7. The spatial gain of the MF beamformer for modes 1 to 40 for an array tilt of .126 degrees that corresponded to ≈ 3 m displacement of the shallowest hydrophone (relative to the deepest hydrophone). The modes experience a loss of less than .2 dB.

estimation error in the order of 1.5 m and a tilt of approximately 3 m. Section IV showed that the estimation errors in the EOF estimation scheme cause an average timing error of 1 ms and a loss of less than 0.2 dB in the spatial gain of the MF beamformer. This shows that the presented estimation scheme works well to recover the missing navigation data with only a minor degradation for mode processing. However, the EOF estimator is not applicable to cases when the navigation data for both the halves of the array are missing. An interpolation method across time would be needed to estimate navigation dropouts across the entire array. A model describing the array motion across time would be needed to design the interpolator across time. A future publication will address array element localization in time.

ACKNOWLEDGEMENTS

The LOAPEX and SPICE04 experiments were funded by the Office of Naval Research (ONR). Tarun Chandrayadula's work was supported by an ONR Ocean Acoustics Graduate Traineeship Award (N00014-06-1-2033). Kathleen Wage's work was supported by an ONR Young Investigator Award (N00014-05-1-0639). The authors acknowledge the significant contributions of the North Pacific Acoustic Laboratory Group, particularly Peter Worcester, Matthew Dzieciuch, Jim Mercer, Bruce Howe, and Rex Andrew, to the design and implementation of the SPICE04 and LOAPEX experiments.

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