The Signals and Systems Concept Inventory

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INTRODUCTION

The *Signals and Systems Concept Inventory* (SSCI) originated in our desire to quantitatively assess the changes in student learning when we adopted active and cooperative learning (ACL) in our undergraduate signals and systems (S&S) courses. These courses are a staple of electrical engineering curricula and are generally taught in the sophomore or junior year. Our interest in ACL for these upper division courses arose when UMass Dartmouth's participation in the NSF-funded Foundation Coalition (FC) [1] prompted a redesign of the integrated freshman year curriculum [2]. The discussions leading to this curriculum revision exposed us to the rich body of physics pedagogical research supporting the use of ACL [3, 4]. Intrigued, we incorporated ACL methods into our S&S courses in the late 1990's. Our pedagogical instincts and student evaluations suggested that students were learning more when we used ACL, but we coveted the quantitative confirmation that Hestenes *et al.*'s *Force Concept Inventory* (FCI) provided for the physics reformers [5]. In the absence of a comparable instrument to the FCI for S&S, we set out to write one.

Our interest in writing the SSCI coincided with a new initiative in Fall 2000 within the FC to promote concept inventories for upper division engineering courses. The FC funded our initial development of the SSCI during late 2000, and version 1.0 of the continuous-time (CT) SSCI was tested at UMass Dartmouth (UMD) and George Mason University (GMU) during the Spring of 2001. The data obtained during these tests guided our revision of the CT instrument to produce version 2.0 and also the first version of the discrete-time (DT) SSCI. In addition to UMD and GMU, the revised instruments were tested at MIT, Old Dominion University, Rose-Hulman Institute of Technology, the U.S. Air Force Academy, and the U.S. Naval Academy during the 2001–2003 period. By the expiration of the FC seed funding in May 2003, the SSCI had been administered to more than 600 students by a dozen instructors. In September of 2005, the NSF Division of Undergraduate Education awarded us additional funding under the Assessment of Student Achievement (ASA) program to refine, validate and then disseminate the SSCI. This new project includes a development team of twelve S&S faculty using the SSCI and providing feedback to the developers, described in more detail below. At the time of this writing, more than 30 faculty have administered the SSCI to over 1400 students.

This paper collects and synthesizes information about the SSCI previously published in a number of different journals and conference proceedings [6, 7, 8]. The target audience of this paper is STEM faculty interested in developing new concept inventories for their own core classes, whereas the publications cited above are targeted at electrical engineering faculty teaching S&S courses. We hope to provide a useful perspective on the development, testing and application of a concept inventory for an upper division core course. This paper also presents new material on our expert peer review process to establish content validity and updates our data employing the SSCI to quantify gains in students' conceptual understanding due to instruction, and to identify stubborn misconceptions held by students.

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THE SIGNALS AND SYSTEMS CONCEPT INVENTORY INSTRUMENT

Signals and Systems (S&S) courses are nearly universal core requirements in the second or third year of electrical engineering curricula. S&S courses teach students to develop abstract mathematical models to represent linear and time-invariant systems, and then to use these models to understand how such systems behave. Many students struggle with this level of abstraction. They find it hard to master using the same techniques to model, for example, a circuit, a shock absorber and a loudspeaker system. Moreover, mastering the techniques taught in S&S courses requires fluency with calculus and differential equations.

In developing the SSCI, we identified five key concepts that all S&S students need to master. These concepts are linearity and time-invariance, convolution, filtering, transform representations (e.g., Laplace and Fourier transforms), and background mathematics. Additionally, sampling is a key concept in any DT S&S course. There are separate CT and DT versions of the SSCI. Each consists of 25 multiple choice questions covering the concepts listed above. The incorrect answers, or distractors, for each question capture common student misunderstandings, which we label misconceptions. It is worth noting that the term *misconception* can have different connotations in the science education literature. To physics educators such as Scherr, a misconception represents “a coherent framework of ideas that are stably present in students minds and present obstacles to instruction” [9]. Modell et al. use the term rather broadly to mean “conceptual and reasoning difficulties”, though they also comment that the vocabulary used in the literature on misconceptions can be confusing since different labels are used to describe similar ideas [10]. Building on the work of diSessa [11], Nasr et al. think of a concept as being composed of a set of knowledge building blocks or “primitives”, and they indicate that misconceptions occur when students combine these primitives in the wrong way or apply a primitive in the wrong context [12, 13]. The papers by Nasr et al. are particularly relevant to the SSCI because they focus on conceptual understanding of signals and systems topics, albeit in the context of aeronautical (rather than electrical) engineering. In this article, we use the term misconception to mean incorrect or incomplete understanding, similar to both Modell and Nasr.

One of the challenges in developing a concept inventory for abstract material such as S&S is to develop good conceptual questions that probe students’ conceptual understanding, and not simply their ability to carry out rote computations. Given the high level of mathematical sophistication required for S&S, some colleagues were skeptical whether it was possible to write conceptual questions in the same spirit as the FCI. Our best definition of a conceptual question is that when the student understands the concept being tested, they can choose the correct answer without computing anything. A good conceptual question contains few numbers, so that students who do not understand the concept have nothing to plug into memorized formulae. The journal article [8] describing the SSCI’s development contains several sample questions from the CT and DT exams.

An important component of the expanded SSCI project under the NSF ASA program is the development team. This team consists of S&S instructors from ten schools in addition to the PI’s schools: Binghamton Univ., Duke Univ., Embry-Riddle Aeronautical Univ., Marquette Univ., Notre Dame, Rice Univ., Rose-Hulman Inst. of Tech., Univ. of California Berkeley, Univ. of Texas-El Paso, and Univ. of Wyoming. These instructors employ the SSCI in their classes in a pretest/posttest protocol. They provide this data, along with linked demographic and academic data to the SSCI project. The linked exam scores and demographic data will allow us to establish construct validity for the SSCI. The exam data from this pool of schools will provide an important baseline on student performance on the SSCI, and help us to characterize common and persistent student misconceptions. The next section presents highlights of the data analysis to date. The development team also meets annually to provide feedback to the SSCI authors on the exam questions, as well as proposed new questions or distractors. These meetings provide expert review to establish content validity for the SSCI, as described below.

OVERVIEW OF SSCI RESULTS

Concept inventories (CI's) can assess student achievement in several ways. First, CI's can measure students' gain in understanding in a class when comparing average scores for the same test administered at the beginning and end of a course (pretest and posttest). Second, detailed analysis of answers to the multiple choice questions provides valuable insights into student misconceptions. Third, correlating CI scores with other factors, such as grades in prerequisite courses, quantifies how students' prior preparation affects their ability to learn new material. Finally, CI's provide a fertile starting point for interviews probing student understanding and misconceptions. To encourage STEM faculty developing CI's to pursue these research directions, this section illustrates the first two of these applications and describes the interviews using examples from the SSCI study. For an example of correlation analysis, see the recent journal article [8].

As noted in the introduction, research by the physics community on the effectiveness of ACL methods motivated the development of the SSCI. Consequently, our analysis of student learning gains as a function of instructional mode derives from Hake's compelling comparison of the learning gains for traditional physics instruction versus ACL-based instruction. As a metric for student learning, Hake defines the normalized gain $\langle g \rangle$:

$$\langle g \rangle = \frac{\text{post-pre}}{\text{100-pre}}, \tag{1}$$

where the pretest and posttest values are the averages for the course, computed using the set of students who took both tests. Based on an analysis of 62 Newtonian physics courses, Hake concludes that the 14 traditional lecture courses achieved normalized gain $\langle g \rangle = 0.23 \pm 0.04$, while 48 ACL courses achieved a significantly higher $\langle g \rangle = 0.48 \pm 0.14$. The SSCI study reveals a similar pattern in the gain for traditional and ACL courses. The average gain for 16 traditional S&S courses is $\langle g \rangle = 0.22 \pm 0.07$ and the average gain for 16 ACL S&S courses is $\langle g \rangle = 0.39 \pm 0.06$. Figure 1 displays the gain data for the 32 courses in the SSCI study using the format suggested by Hake. The plot shows the raw difference between the pretests and posttests versus pretest score. Note that the ACL courses cluster in the medium gain region, which Hake defined as gains between 0.3 and 0.7.

While gain statistics are an indicator of students' overall performance, computing the difficulty index for a CI provides more detailed information about which concepts are the hardest for students to master. The difficulty index is defined as the percentage of students answering a question correctly. Figure 2 shows the difficulty index for the CT-SSCI computed using pretest and posttest data from a pool of 445 students. The labels on the figure highlight the results for selected concepts. Students perform best on the four questions related to background mathematics. This probably reflects the numerous opportunities students have to practice their basic math skills during any S&S course. Figure 2 also indicates results for questions on core SSCI topics: time/frequency relationships (*i.e.,* transform representations), convolution, and filtering. While there is substantial gain between pre and post for these three questions, it is disconcerting that, even on the posttest, less than 60% of students are responding correctly.

Comparison of the pre and post difficulty indexes also identifies questions for which student performance is worse at the end of the semester than at the beginning, perhaps indicating that a little knowledge can be a dangerous thing. For instance, note that Figure 2 shows that fewer students answer Question 18 correctly on the posttest than the pretest. They appear to be guessing on the pretest since the percentage correct is just below the 25% predicted by chance. Analysis of the posttest answers indicates that most students are choosing a distractor that is partially correct. Question 18 displays a set of pole-zero plots and asks students which could correspond to real systems. 75% of students choose the distractor indicating that they believe real systems must have all real poles and zeros, neglecting that systems with complex conjugate poles and zeros can also be real. The SSCI results indicate that this misconception is very persistent, meaning that it is resistant to instruction. We define the persistence of a distractor by the fraction of the students who choose that same distractor on both the pretest and posttest [8]. The main distractor for Question 18

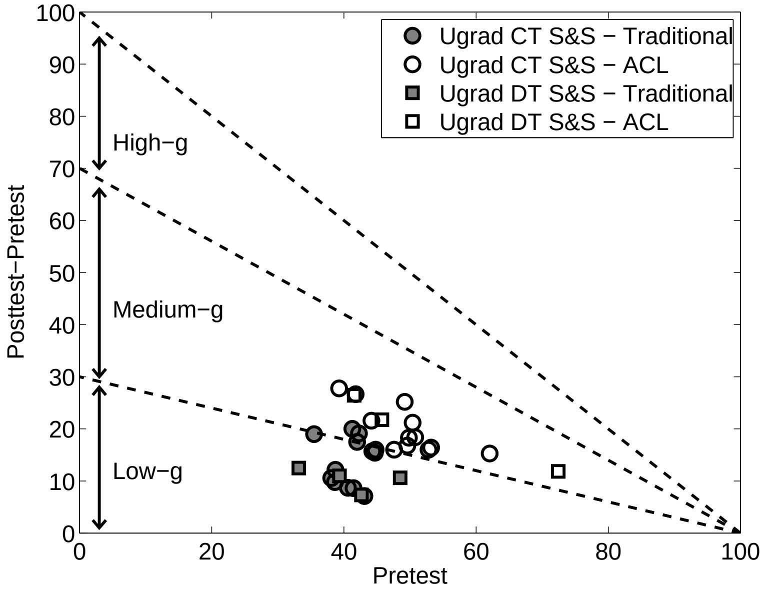

Figure 1: Comparison of raw gain versus pretest score for 32 S&S courses. Each point represents a single course. The abscissa is the pretest score and the ordinate is the raw gain, defined as the posttest average minus the pretest average for the course. The low, medium and high gain regions are those defined by Hake [4]. 16 of the 32 courses employed ACL methods.

is chosen by 42% of students on both the pre and posttests, in contrast to the 6.25% predicted by chance if students were only guessing on both the pretest and posttest. This suggests that S&S courses do little to alter students' misconceptions about where the poles and zeros of a real system must lie. Several other distractors on the CT-SSCI are significantly persistent. Analysis of persistence is useful because it indicates where new explanations and exercises are most needed to help students overcome resistant misconceptions.

Quantitative analysis of multiple-choice test data from a CI does not provide a complete picture of students' thought processes. Conducting student interviews based on CI questions gives additional insight into how students approach the subject, and may reveal unanticipated misconceptions or mental models used by students. For these reasons, the SSCI project includes funding to interview students about their thought processes in solving SSCI questions. The first round of interviews was conducted in spring 2006 [7], and additional interviews are ongoing. To date the interview questions have focused on concepts related to frequency-selective filtering. To understand filtering students must also have a good understanding of sinusoidal signals, linear time-invariant systems, and the relationship between time and frequency. The CT-SSCI includes four questions on these basic concepts. The interviews revealed that students find the Fourier transform to be difficult, and that they have misconceptions about the roles of the magnitude and phase of a system's frequency response. One unexpected outcome of the interviews was the discovery that the connotations of the everyday use of the term "filter" (*e.g.,* coffee filter, spam filter) may limit students' concept of how filters work in S&S. There is also some indication that the heavy emphasis placed on ideal lowpass filters as examples in S&S courses may encourage students to overgeneralize how filters work, leading to problems when they try to apply the same ideas in other contexts.

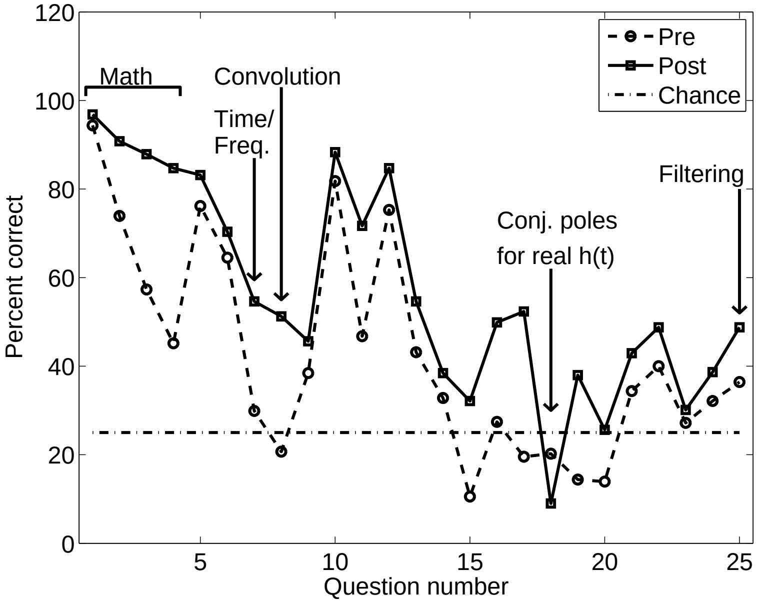

Figure 2: Pretest and posttest difficulty index for the CT-SSCI based on data from 445 students. The dashed line at 25% indicates the expected result for random guessing with four possible choices.

EXPERT REVIEW FOR CONTENT VALIDITY

As part of the current phase of SSCI development, we are holding day-long meetings with the development team consisting of faculty users of the SSCI. The first goal is to obtain feedback about the questions on the SSCI since there is variation among institutions and faculty in terms of course structure, course sequencing, and course content in S&S. The second goal of the meetings is to ensure the content validity of the concept inventory. We want to know if the CT and DT exams were covering the content experts in the field felt was important and if the content was covered accurately. At the first team meeting, eleven professors from different institutions attended (in addition to the authors). All of them are experienced S&S instructors. Most had already used the SSCI with their students, and the remainder had committed to using the SSCI in their classes during the 2006–2007 academic year. The development team members come from a variety of institutions, ranging from small, private universities to large, public universities. All have an interest in the teaching aspects of their work, and a few are in primarily teaching positions.

In the first part of the meeting, we asked the development team to identify the three hardest concepts and the three most important concepts in S&S. After they compiled lists individually, we held a group discussion of the topics. The two most frequently identified as difficult concepts were convolution and the relationship between the poles of the system function and time domain behavior. More generally, these difficult concepts are related to students' understanding of functions and transformation of functions in the context of S&S. Under important concepts, faculty also identified convolution but included frequency and time relationships, as well as the examination of non-linear functions. As an example, properties and transformations of periodic functions play a significant role in S&S. Complex number representations, Fourier transforms, Fourier series, non-linearity, and linear time invariance were also highlighted. Overall, the main challenge for students is the application and integration of mathematical topics in the context of signals and systems. In the second part of the day-long meeting, the development team completed either the CT or DT SSCI in order to help them understand students' experience of the exam, and to identify any potential improvements in wording or formatting of the questions.

The expert review of the exam as well as the identification of both difficult and important concepts confirmed that the SSCI was covering topics important to the discipline. One aspect of the discussion focused on how important it is to include symbolic representations of functions in the questions or whether questions could be stated without symbolic representations. As identified by at least one student interviewee, the symbols can be a point of difficulty for students and their use may vary. The symbols used may add another layer of abstraction to an abstract topic. Another issue was whether discrete time signals were covered in parallel with continuous time signals or whether the material was covered in two separate courses. While there are relationships between the concepts, departments organize the sequencing differently.

CONCLUSION

To date the SSCI project has obtained several important results. First, the analysis of gains on the SSCI reveals a striking similarity to Hake's results for the FCI. For a data set consisting of over 1300 S&S students, we found that on average, students in ACL courses gain roughly twice as much conceptual understanding as those in traditional lecture courses ($\langle g \rangle = 0.39$ vs. $\langle g \rangle = 0.22$). Second, an analysis of students' answers highlighted several persistent misconceptions that students have about key S&S topics. Third, our pilot interview study found that students bring unexpected preconceptions about filtering into signals and systems courses, probably based on commonplace uses of the word ”filter.” Finally, the faculty development team has provided significant feedback that will be used to revise the SSCI exams in the upcoming year.

The SSCI project is an ongoing effort funded by NSF's ASA program through 2008. In addition to the types of analysis and peer review described in this paper, we are also investigating the reliability of the SSCI and checking for gender and racial bias using student data provided by the development team. The goal of the SSCI project is to provide faculty with a reliable, validated instrument that can be used for formative feedback, as well as for accreditation assessment. It is exciting to note that several instructors are currently using the SSCI to assess the effects of different instructional methods, such as project-based courses [14, 15] and graphical vs. text-based programming methods [16].

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Kathleen E. Wage received her Ph.D. in electrical engineering from the Massachusetts Institute of Technology/Woods Hole Oceanographic Institution Joint Program. Her research interests include array processing, underwater acoustics, and ocean acoustic tomography. Dr. Wage received an Office of Naval Research (ONR) Young Investigator Award in 2005 and an ONR Ocean Acoustics Entry-Level Faculty Award in 2002. She has won several teaching awards, including the Outstanding Teaching Award from George Mason University's School of Information Technology and Engineering and the Harold L. Hazen Teaching Award from the MIT EECS department.

John R. Buck received his Ph.D. from the Massachusetts Institute of Technology and Woods Hole Oceanographic Institution Joint Program in Electrical and Oceanographic Engineering. His research focuses on applications of signal processing and information theory to underwater acoustics and animal bioacoustics. He is the co-author of two popular textbooks in signal processing: *Discrete-Time Signal Processing, Second Edition* by Oppenheim and Schafer with Buck, and *Computer Explorations in Signals and Systems Using Matlab (TM), Second Edition* by Buck, Daniel and Singer. The IEEE Education Society awarded him the 2005 Mac Van Valkenburg Early Career Teaching Award. His other honors include an NSF CAREER award, an ONR Young Investigator Award, a Fulbright Senior Scholar Fellowship, and the MIT Goodwin Medal for conspicuously effective teaching by a graduate instructor.

Margret Hjalmarson received her Ph.D. in mathematics education and her M.S. in mathematics from Purdue University. Her current research interests are in engineering education, mathematics curriculum, and assessment. She has also researched design research methods for mathematics education from a models and modeling perspective.

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