Homework #6 Solutions

1. Roth & Kinney – problem 1.37
2. Roth & Kinney – problem 1.5, parts (a) and (b)
3. Roth & Kinney – problem 1.7, parts (a), (c), and (e)
4. Roth & Kinney – problem 1.34, parts (a), (b), and (c)
5. Roth & Kinney – problem 1.25
6. Roth & Kinney – problem 4.36
7. Roth & Kinney – problem 4.38
8. Roth & Kinney – problem 4.39
9. Roth & Kinney – problem 4.40
#1. Roth & Kenney - prob. 1.37

(a) 1's Complement Binary Numbers.

(i) 0000000 → 1's comp = 1111111
decimal = 0
decimal = -0.

(ii) 1111111 → 1's comp = 0000000
decimal = -0

decimal = 0

(iii) 00110011 → 1's comp = 11001100
decimal = 51

decimal = -51

(iv) 1000000 → 1's comp = 0111111
decimal = -63*
decimal = 63

* This value was determined only after the magnitude of the corresponding value was determined.
(b) 2's Complement Binary Numbers.

(i) 0000000 → 2's Comp = 1111111 + 1 = 0000000
   decimal = 0

(ii) 1111111 → 2's Comp = 0000000 + 1 = 0000001
    decimal = -1

(iii) 00110011 → 2's Comp = 11001101 + 1 = 11001110
      decimal = 51

(iv) 1000000 → 2's Comp = 0111111 + 1 = 1000000
     decimal = -64
#2. Add, Subtract, and Multiply in binary.  

(a) \[\begin{array}{c}
1111 \\
+ 1010 \\
\hline
11001
\end{array}\]  

(b) \[\begin{array}{c}
110110 \\
+ 011101 \\
\hline
1010011
\end{array}\]  

\[\begin{array}{c}
0110110 \\
\times 1010 \\
\hline
0000000 \\
11110000 \\
\hline
10010110
\end{array}\]

**note:** the above addition, subtraction, and multiplication operations were carried out assuming that the binary numbers were **unsigned** and there was no specific number of bits that we were restricted to.
#3. Roth & Kinney - problem 1.7
For 2's Complement representation.

(a) \(21_{10} = 010101_2\) (using 6 bits)
\(11_{10} = 001011_2\) (using 6 bits)

\[
\begin{align*}
010101 & \quad \text{positive} \\
+ 001011 & \quad \text{positive} \\
\hline
100000 & \quad \text{negative}
\end{align*}
\]

OVERFLOW - addition of 2 positive numbers resulted in a negative sum.

(b) \(-25_{10} \Rightarrow +25_{10} = 011001_2\)
\[-25_{10} = 100110 + 1 = 100111_2\] (2's Comp.)
\(18_{10} = 0100010_2\)

\[
\begin{align*}
\overline{100111} \\
+ 0100010 \\
\hline
111001 & \quad \text{negative number}
\end{align*}
\]

NO OVERFLOW.

\[
\begin{align*}
111001 & \Rightarrow 2's \text{ comp} = 000110 \\
+ 1 \\
\hline
000111_2 = +7_{10}
\end{align*}
\]

\(-111001 = -7\) in 2's Complement Binary using 6 bits.
(e) \(-11_{10} \Rightarrow +11_{10} = 001001_{2}\)

\[ -11_{10} = 110100 + 1 = 110101_{2} \text{ (2's Comp.)} \]

\(-21_{10} \Rightarrow +21_{10} = 010101_{2}\)

\[ -21_{10} = 101010 + 1 = 101011_{2} \text{ (2's Comp.)} \]

\[
\begin{array}{c}
110101 \\
+ 101011 \\
\hline
1100000
\end{array}
\]

\(\uparrow\) ignore

\(\text{NO OVERFLOW.}\)
For 1's Complement Representation

(a) \( 21_{10} = 01\ 0101_2 \) (using 6 bits)

\[ 11_{10} = 00\ 1011_2 \] (using 6 bits)

\[
\begin{array}{c}
\text{01 0101}_2 \\
\text{00 1011}_2 \\
\hline
\text{10 0000}
\end{array}
\]

\[ \text{OVERFLOW.} \]

(c) \(-25_{10} \Rightarrow +25_{10} = 01\ 1001_2\]

\[
\begin{array}{c}
\text{18}_{10} = 01\ 0010_2 \\
\hline
\text{10 0110}
\end{array}
\]

\[ \text{NO OVERFLOW} \]

\[ 111000 \rightarrow 1's\ \text{Comp} = 000111_2 = +7_{10} \]

\[ \Rightarrow 111000 = -7 \text{ in 1's Complement Binary using 6 bits.} \]
(e) 

\[-11_{10} \Rightarrow +11_{10} = 00 \ 1011_2\]

\[-11_{10} = 11 \ 0100_2\ (1's\ Comp.)\]

\[-21_{10} \Rightarrow +21_{10} = 01 \ 0101_2\]

\[-21_{10} = 10 \ 1010_2\ (1's\ Comp.)\]

\[11 \ 0100\ \leftarrow \text{negative}\]

\[+10 \ 1010\ \leftarrow \text{negative}\]

\[\overline{1011110}\]

\[\rightarrow 1\]

\[011111\ \leftarrow \text{positive}\]
# ECE 331/3301 - Homework #6 Solutions

#4. Roth & Kenney - problem 1.34

For 1's Complement Representation

(a) $01001 \quad \Rightarrow \quad 01001 \quad (a)$

- $11010 \quad \Rightarrow \quad +00101 \quad (5)$

\[ \underline{01110} \quad (14) \]

\[ \frac{11010}{+00110} \]

\[ \underline{00000} \rightarrow 1 \]

\[ \underline{00001} \]

(b) $11010 \quad \Rightarrow \quad 11010$

- $11001 \quad \Rightarrow \quad +00110$

\[ \underline{100000} \rightarrow 1 \]

\[ \frac{00001}{\text{NO OVERFLOW}} \]

(c) $10110 \quad \Rightarrow \quad 10110 \leftarrow \text{negative}$

- $01101 \quad \Rightarrow \quad +10010 \leftarrow \text{negative}$

\[ \underline{101000} \rightarrow 1 \]

\[ \underline{01001} \leftarrow \text{positive} \]
#4. (continued)

For 2's Complement Representation.

(a) \[ 01001 - 11010 \rightarrow 01001 + 00110 = 01111 \]

\[ \text{NO} \Rightarrow \text{OVERFLOW}. \]

(b) \[ 11010 - 11001 \rightarrow 11010 + 00111 = 00001 \]

\[ \text{NO} \Rightarrow \text{OVERFLOW}. \]

\[ \text{Ignore} \]

(c) \[ 10110 - 01101 \rightarrow 10110 + 10011 = 101001 \]

\[ \text{OVERFLOW}. \]

\[ \text{Ignore} \]

\[ \neg \text{negative} \]

\[ \neg \text{positive} \]
#5. Roth & Kinney - problem 1.25

4-3-2-1 Weighted Code.

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9154 = 1110 0001 1001 1000
6. Roth & Kinney – problem 4.36  

**EXCESS-3 CODE**

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(a) \( X = \chi m(1, 2, 3, \ldots, 14, 15) \)

\( Y = \chi m(0, 7, 11, 13, 14, 15) \)

\( Z = \chi m(0, 3, 5, 6, 9, 10, 12, 15) \)

(b) \( Y = M \bar{\Gamma}(1, 2, 3, 4, 5, 6, 8, 9, 10, 12) \)

\( Z = M \bar{\Gamma}(1, 2, 4, 7, 8, 11, 13, 14) \)
Roth & Kinney – problem 4.38

OUTPUT = \((4 \times \text{INPUT}) + 1\)

*note: the input represents a BCD digit – therefore, only values 0-9 are valid.

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S = 0; T = 0

U = A + A'B(C+D)

V = A + A'(B'CD + BC'D')

**Using** \(X + X'Y = X + Y\), (Boolean Thm) \(U\) and \(V\) can be simplified.

\[Z = 1\]

\[Y = V = A + B'CD + BC'\]

\[X = B'C'D + BD'\]

\[W = B'CD + BCD\]

Spring 2011
#8. Roth & Kenney - problem 4.39

- Adder circuit to add a 4-bit signed binary number to a 5-bit signed binary number.
- Negative numbers are represented using 2's Complement.
  - Must sign-extend the 4-bit binary number to make it a 5-bit binary number.
  - Since 2's Complement is used to represent negative numbers, no additional logic (i.e. gates) is required to carry out the addition.

```
A4
A3
FA4
C4
S4
C5
FA3
A2
B2
FA2
C3
S3
FA1
A1
B1
FA0
C1
S1
FA0
A0
B0
S0
```

A = 5-bit number
B = 4-bit number
- B is sign extended by connecting B3 to FA3 and FA4

*note: C0 (the carry-in for FA0) is not shown.*
Half Adder:

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2's Complement of $X = X' + 1$

- Add one.
- Bit-wise complement (NOT gate)

$X^* = 2's$ Comp of $X$.

Binary number $\rightarrow$ HA

2's Complement of Binary number $\rightarrow$  

Diagram of half adders and carry generation.