Please read the homework notes given on the previous assignments.

Reflection Questions for PS6 (5 points)

(a) Compare your answers to the problems on PS4 to the solutions posted on the Blackboard site. Summarize what you learn from this comparison. What problems did you solve correctly? What problems did you solve incorrectly? Do you now understand how to solve the problems? Do you have questions about how the solutions were implemented?

(b) Write a one-paragraph summary of what you learned from the problems in this assignment. Your summary should include a brief description of the key concepts required to solve the problems. Comment on any problems you found difficult to solve and why they were difficult.

(c) What (if any) questions do you still have about the material covered on this problem set?

Problem PS6-1
Sketch the spectra of the following signals:

(a) \( v(t) = 1 + \cos(2\pi(5)t) + \cos(2\pi(15)t) \)

(b) \( x(t) = v(t) \cos(2\pi(100)t) \)

(c) \( y_1(t) = 2x(t) \cos(2\pi(100)t) \)

(d) \( y_2(t) = x(t) \cos(2\pi(50)t) \)

(e) \( y_3(t) = \frac{1}{2}x(t) \cos(2\pi(25)t) \)

Problem PS6-2
Dilbert and his colleagues Wally and Alice are discussing ways to recover an amplitude modulated signal. The recovery process is called demodulation. The three proposed demodulation systems described below all use a processor called a lowpass filter. You can think of a lowpass filter as a device that keeps the low frequencies and throws away the high frequencies. Specifically, it keeps all frequencies less than the filter cutoff frequency. The designer of the filter specifies its cutoff frequency.

The signal that Dilbert and his colleagues are working with is \( x(t) = v(t) \cos(2\pi f_c t) \) where \( f_c = 100 \text{ Hz} \) and \( v(t) = 1 + \cos(2\pi(25)t) \). Sketch the spectra of \( v(t) \) and \( x(t) \).

Dilbert suggests that the signal \( v(t) \) can be recovered from \( x(t) \) by multiplying \( x(t) \) by \( \frac{1}{2} \cos(2\pi(25)t) \) and then passing it through a lowpass filter with a cutoff frequency of 200 Hz.

Wally suggests that the signal \( v(t) \) can be recovered from \( x(t) \) by multiplying \( x(t) \) by \( 1 \cos(2\pi(50)t) \) and then passing it through a lowpass filter with a cutoff frequency of 100 Hz.

Alice suggests that the signal \( v(t) \) can be recovered from \( x(t) \) by multiplying \( x(t) \) by \( 2 \cos(2\pi(100)t) \) and then passing it through a lowpass filter with a cutoff frequency of 50 Hz.

Who do you agree with? Dilbert, Wally, Alice? Provide sketches to support your solution. You should clearly illustrate why one solution works and why the other solutions do not work.
Problem PS6-3
Suppose that the FCC has licensed 3 radio stations to broadcast with center frequencies of 1 MHz, 2 MHz, and 3 MHz, respectively. These radio stations are allowed to use up to ±200 kHz of bandwidth around their center frequencies. To give these radio stations some protection from unwanted interference, the FCC has said that no one can use the spectrum within ±300 kHz of the three center frequencies, i.e., there should be a quiet zone that contains no broadcast signals that extends 100 kHz on either side of each station’s designated band. The unregulated spectrum between the stations is free to be used by anyone.

You have been hired by a local startup company called Mason Communications (MC). MC is developing a simple comms link that broadcasts amplitude modulated signals. The message signal it will broadcast is \( v(t) \) defined below:

\[
v(t) = 1 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t)
\]

(The \( A_k \)'s will be designed to carry a message; you do not need to worry about this aspect of the problem.) The AM modulated signal that is sent out will be \( x(t) = v(t) \cos(2\pi f_c t) \). Your boss at MC has hired you to determine whether the parameters they have chosen for the system will guarantee compliance with the FCC regulations. The possible parameter sets your boss suggested are as follows

- **Set 1**: \( N = 100 \), \( f_0 = 1 \) kHz, \( f_c = 1.5 \) MHz.
- **Set 2**: \( N = 10 \), \( f_0 = 500 \) Hz, \( f_c = 2.25 \) MHz.
- **Set 3**: \( N = 5 \), \( f_0 = 10 \) kHz, \( f_c = 3.4 \) MHz.
- **Set 4**: \( N = 250 \), \( f_0 = 1 \) kHz, \( f_c = 2.5 \) MHz.

Which if any of these sets of parameters comply with the FCC regulations? Justify your answers. If none of the sets work, design a new set of parameters that does meet the regulations.

Problem PS6-4 (Old ECE 201 exam problem)
Consider the 4 spectrograms shown on the following page. Match each spectrogram to one of the equations below. For each match you make provide a short justification. **Answers without justifications will be assumed to be guesses and receive no credit.**

Choices:

- \( x_1(t) = \cos(-2\pi(125)t^2 + 2\pi(2000)t) \)
- \( x_2(t) = \cos(2\pi(125)t^2 + 2\pi(2000)t) \)
- \( x_3(t) = \cos(-2\pi(500)t^2 + 2\pi(4500)t) \)
- \( x_4(t) = \cos(2\pi(500)t^2 + 2\pi(4500)t) \)
- \( x_5(t) = (1 + \cos(2\pi(500)t)) \cos(2\pi(3500)t) \)
- \( x_6(t) = \cos(2\pi(500)t) \cos(2\pi(3500)t) \)
- \( x_7(t) = \cos(2\pi(3000)t) + \cos(2\pi(2500)t) \)
- \( x_8(t) = \cos(2\pi(3000)t) \cos(2\pi(2500)t) \)