The purpose of this project is to explore the connection between the poles of a system and its time domain behaviour.

Your report for this project will consist of all the analytical (i.e., pencil/paper) work, Matlab plots and code, and relevant explanations. A list of guidelines for preparing the lab report is be posted on the ECE 220 website. Each student must do his or her own work on this project, however you may ask other students or any of the teaching staff for advice. As stated in the guidelines given in the ECE 220 course information packet, you should identify any students you talk to about the project.

1 Analytical Work

Consider the second-order system defined by the system function \( H(s) \) given below:

\[
H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.
\]  

The parameter \( \zeta \) (zeta) is known as the damping ratio and the parameter \( \omega_n \) is known as the undamped natural frequency of the system.

(a) Determine an analytical expression for the two poles of this system in terms of \( \zeta \) and \( \omega_n \). \textit{Hint: use the quadratic formula!}

For the remainder of this project, assume that \( \omega_n = 1 \). You will investigate how the parameter \( \zeta \) affects the system response.

(b) Answer the following questions about the pole locations.

(i) What values of the parameter \( \zeta \) result in the two poles being real (the imaginary part is zero)?
(ii) What values of the parameter \( \zeta \) result in the two poles being imaginary (the real part is zero)?
(iii) What values of \( \zeta \) yield two equal poles?
(iv) What values of \( \zeta \) yield a stable system? Recall that a causal LTI system is stable if its poles are in the left half plane.

(c) Now consider the form of the step response of the system, \textit{i.e.}, the output of the system when the input is equal to \( u(t) \). What is \( Y(s) \) (Laplace transform of the output) when \( u(t) \) is the input? Using partial fraction expansion of \( Y(s) \) followed by an inverse transform, you can calculate the output of the system. Determine the form of the step response for the following cases:

(i) the poles are real and unequal
(ii) the two poles are equal
(iii) the two poles are complex conjugates

Note: you do \textbf{not} need to compute the coefficients of the partial fraction expansion! All that is required is to determine the form of the time-domain solution.

(d) Comment on what these solutions should look like. Do they oscillate or not? Will they eventually reach a constant value? If so, what will this value be equal to?
2 Matlab Simulations

In this part you will consider the following values for the damping ratio: $\zeta = 0, 0.1, 0.2, 0.5, 1.0, 2.0, 5.0, -0.2, -2.0$.

(a) For each value of $\zeta$, plot the location of the poles of the system. An example of a pole plot is shown in Figure 1. This type of plot displays the real part of the pole on the x-axis and the imaginary part of the pole on the y-axis. You may use the Matlab function `pzmap` to make the pole plot or simply calculate the poles using the `roots` command and then plot their locations in the complex plane using the `plot` command. For example if you stored the poles in the `p` vector, you could plot them using the command `plot(real(p),imag(p),'x')`. If you use the `pzmap` command, you will need to define the system using the `tf` command described in Lab 3.

(b) For each value of $\zeta$, compute the step response of the system out to time $t = 60$ using the `step` command. Note that the function `step` allows you to specify the final time as an argument. Alternatively, you could give it a vector of times, e.g., `t=0:0.01:60`. You may get smoother-looking plots if you specify a vector of times. Compare the step responses and the pole plots. For each value of $\zeta$ discuss the following:

- Does the step response settle to a final value?
- If it does settle, how much time does it take to do so?
- Does the step response overshoot the final value? by how much?
- How much time does it take for the output to reach to the final value? This is the rise time. (If it oscillates around the final value, the rise time is equal to the first time it reaches the final value.)
- Relate your answers to the pole plots. Based on the pole locations, could you predict the behaviour of the step response?