Chapter 2

Mobility

2.1 (a) The mean free time between collisions using Equation (2.2.4b) is

\[
\mu_n = \frac{q \tau_{mn}}{m_n} \rightarrow \tau_{mn} = \frac{\mu_n m_n}{q} = 2.85 \times 10^{-13} \text{ sec}
\]

where \( \mu_n \) is given to be 500 cm\(^2\)/Vsec (= 0.05 m\(^2\)/Vsec), and \( m_n \) is assumed to be \( m_0 \).

(b) We need to find the drift velocity first:

\[
v_d = \mu_n E = 50000 \text{ cm/sec}.
\]

The distance traveled by drift between collisions is

\[
d = v_d \tau_{mn} = 0.14 \text{ nm}.
\]

2.2 From the thermal velocity example, we know that the approximate thermal velocity of an electron in silicon is

\[
v_{th} = \sqrt{\frac{3kT}{m}} = 2.29 \times 10^7 \text{ cm/sec}.
\]

Consequently, the drift velocity \( (v_d) \) is \((1/10)v_{th} = 2.29 \times 10^6 \text{ cm/sec}\), and the time it takes for an electron to traverse a region of 1 \( \mu \text{m} \) in width is

\[
t = \frac{10^{-4} \text{ cm}}{2.29 \times 10^6 \text{ cm/sec}} = 4.37 \times 10^{-11} \text{ sec}.
\]

Next, we need to find the mean free time between collisions using Equation (2.2.4b):

\[
\mu_n = \frac{q \tau_{mn}}{m_n} \rightarrow \tau_{mn} = \frac{\mu_n m_n}{q} = 2.10 \times 10^{-13} \text{ sec}
\]

where \( \mu_n \) is 1400 cm\(^2\)/Vsec (=0.14 m\(^2\)/Vsec, for lightly doped silicon, given in Table 2-1), and \( m_n \) is 0.26\( m_0 \) (given in Table 1-3). So, the average number of collision is
\[
\frac{t}{\tau_{mn}} = 207.7 \text{ collision} \Rightarrow 207 \text{ collisions}.
\]

In order to find the voltage applied across the region, we need to calculate the electric field using Equation (2.2.3b):

\[
v_d = -\mu_n E \Rightarrow E = \frac{v_d}{\mu_n} = \frac{2.29 \times 10^6 \text{ cm/ sec}}{1400 \text{ cm}^2 / \text{ V sec}} = 1635.71 \text{ Vcm}^{-1}.
\]

Then, the voltage across the region is

\[
V = E \times \text{width} = 1635.71 \text{ Vcm}^{-1} \times 10^{-3} \text{ cm} = 0.16V.
\]

2.3 (a)

(b) If we combine \( \mu_1 \) and \( \mu_2 \),

\[
\text{Log}[\mu]
\]
The total mobility at 300 K is
\[
\mu_{\text{TOTAL}}(300\, K) = \left(\frac{1}{\mu_1(300\, K)} + \frac{1}{\mu_2(300\, K)}\right)^{-1} = 502.55\, cm^2/V\, sec.
\]

(c) The applied electric field is
\[
\varepsilon = \frac{V}{l} = \frac{1V}{1mm} = 10V/cm.
\]

The current density is
\[
J_{\text{drift}} = q\mu_n n\varepsilon = q\mu_n N_d\varepsilon = 80.41A/cm^2.
\]

**Drift**

2.4 (a) From Figure 2-8 on page 45, we find the resistivity of the N-type sample doped with 1×10¹⁶ cm⁻³ of phosphorous is 0.5 Ω·cm.

(b) The acceptor density (boron) exceeds the donor density (P). Hence, the resulting conductivity is P-type, and the net dopant concentration is \(N_{\text{net}} = |N_d-N_a| = p = 9×10¹⁶\, cm⁻³\) of holes. However, the mobilities of electrons and holes depend on the total dopant concentration, \(N_T=1.1×10¹⁷\, cm⁻³\). So, we have to use Equation (2.2.14) to calculate the resistivity. From Figure 2-5, \(\mu_p(N_T=1.1×10¹⁷\, cm⁻³)\) is 250 cm²/V·sec. The resistivity is
\[
\rho = \frac{1}{\sigma} = \frac{1}{qN_{\text{net}}\mu_p} = \frac{1}{q \times 9 \times 10^{16} \frac{cm^{-3}}{250 cm^2/V sec}} = 0.28\, \Omega\, cm.
\]

(c) For the sample in part (a),
\[
E_c - E_f = kT \ln \left(\frac{N_c}{N_d}\right) = 0.026V \ln \left(\frac{2.8 \times 10^{19} \, \text{cm}^{-3}}{10^{16} \, \text{cm}^{-3}}\right) = 0.21\, eV.
\]
For the sample in part (b),

\[ E_f - E_c = kT \ln \left( \frac{N_v}{N_{net}} \right) = 0.026V \ln \left( \frac{1.04 \times 10^9 \text{cm}^{-3}}{9 \times 10^6 \text{cm}^{-3}} \right) = 0.12eV \]

\[ E_c \]

\[ 0.12 \text{eV} \]

\[ E_i \]

\[ E_f \]

\[ E_f \]

\[ E_i \]

2.5 (a) Sample 1: N-type □ Holes are minority carriers.
\[ p = n_i^2 / N_d = (10^{10} \text{cm}^{-3})^2 / 10^{17} \text{cm}^{-3} = 10^2 \text{ cm}^{-3} \]

Sample 2: P-type □ Electrons are minority carriers.
\[ n = n_i^2 / N_a = (10^{10} \text{cm}^{-3})^2 / 10^{15} \text{cm}^{-3} = 10^5 \text{ cm}^{-3} \]

Sample 3: N-type □ Holes are minority carriers.
\[ p = n_i^2 / N_{net} = (10^{10} \text{cm}^{-3})^2 / (9.9 \times 10^{17} \text{cm}^{-3}) \approx 10^2 \text{ cm}^{-3} \]

(b) Sample 1: \[ N_d = 10^{17} \text{cm}^{-3} \]
\[ \mu_n(N_d = 10^{17} \text{cm}^{-3}) = 750 \text{ cm}^2/\text{Vsec} \text{ (from Figure 2-4)} \]
\[ \sigma = qN_d \mu_n = 12 \Omega^{-1}\text{cm}^{-1} \]

Sample 2: \[ N_a = 10^{15} \text{cm}^{-3} \]
\[ \mu_p(N_a = 10^{15} \text{cm}^{-3}) = 480 \text{ cm}^2/\text{Vsec} \text{ (from Figure 2-4)} \]
\[ \sigma = qN_a \mu_p = 12 \Omega^{-1}\text{cm}^{-1} \]

Sample 3: \[ N_T = N_d + N_a = 1.01 \times 10^{17} \text{cm}^{-3} \]
\[ \mu_n(N_T = 1.01 \times 10^{17} \text{cm}^{-3}) = 750 \text{ cm}^2/\text{Vsec} \text{ (from Figure 2-4)} \]
\[ N_{net} = N_d - N_a = 0.99 \times 10^{17} \text{cm}^{-3} \]
\[ \sigma = qN_{net} \mu_n = 11.88 \Omega^{-1}\text{cm}^{-1} \]

(c) For Sample 1,

\[ E_c - E_f = kT \ln \left( \frac{N_c}{N_d} \right) = 0.026V \ln \left( \frac{2.8 \times 10^{19} \text{cm}^{-3}}{10^{17} \text{cm}^{-3}} \right) = 0.15eV \]

\[ E_c \]

\[ 0.15 \text{eV} \]

\[ E_f \]

\[ E_i \]

\[ E_i \]

\[ E_f \]
For Sample 2,

\[ E_f - E_v = kT \ln \left( \frac{N_c}{N_a} \right) = 0.026V \ln \left( \frac{1.04 \times 10^{19}}{10^{15}} \text{cm}^{-3} \right) = 0.24 \text{eV}. \]

For Sample 3,

\[ E_c - E_f = kT \ln \left( \frac{N_c}{N_{\text{net}}} \right) = 0.026V \ln \left( \frac{2.8 \times 10^{19}}{9.9 \times 10^{16}} \text{cm}^{-3} \right) = 0.15 \text{eV}. \]

2.6 (a) From Figure 2-5, \( \mu_n (N_d = 10^{16} \text{cm}^{-3} \text{ of As}) \) is 1250 cm\(^2\)/Vs. Using Equation (2.2.14), we find

\[ \rho = \frac{1}{\sigma} = \frac{1}{qn\mu_n} = 0.5 \ \Omega \text{cm}. \]

(b) The mobility of electrons in the sample depends not on the net dopant concentration but on the total dopant concentration \( N_T \):

\[ N_T = N_d + N_a = 2 \times 10^{16} \text{cm}^{-3}. \]

From Figure 2-5,

\[ \mu_n (N_T) = 1140 \text{cm}^2 / \text{Vs} \quad \text{and} \quad \mu_p (N_T) = 390 \text{cm}^2 / \text{Vs}. \]

\( N_{\text{net}} = N_d - N_a = 0 \). Hence, we can assume that there are only intrinsic carriers in the sample. Using Equation (2.2.14),
\[ \rho = \frac{1}{\sigma} = \frac{1}{qn \mu_n + q \mu_p} = \frac{1}{qn \left( \mu_n + \mu_p \right)} \]
\[ = \frac{1}{q \times 1 \times 10^{19} \text{cm}^{-3} \times (1140 + 390) \text{cm}^2 / \text{V sec}}. \]

The resistivity is \( 4.08 \times 10^5 \Omega \cdot \text{cm}. \)

(c) Now, the total dopant concentration \( (N_T) \) is 0. Using the electron and hole mobilities for lightly doped semiconductors (from Table 2.1), we have
\[ \mu_n = 1400 \text{cm}^2 / \text{V sec} \quad \text{and} \quad \mu_p = 470 \text{cm}^2 / \text{V sec}. \]

Using Equation (2.2.14),
\[ \rho = \frac{1}{\sigma} = \frac{1}{qn \mu_n + q \mu_p} = \frac{1}{qn \left( \mu_n + \mu_p \right)} \]
\[ = \frac{1}{q \times 1 \times 10^{19} \text{cm}^{-3} \times (1400 + 470) \text{cm}^2 / \text{V sec}}. \]

The resistivity is \( 3.34 \times 10^5 \Omega \cdot \text{cm}. \) The resistivity of the doped sample in part (b) is higher due to ionized impurity scattering.

2.7 It is given that the sample is \( n \)-type, and the applied electric field \( \varepsilon \) is 1000 V/cm. The hole velocity \( v_{dp} \) is \( 2 \times 10^5 \text{cm/s}. \)

(a) From the velocity and the applied electric field, we can calculate the mobility of holes:
\[ v_{dp} = \mu_p \varepsilon, \quad \mu_p = v_{dp} / \varepsilon = 2 \times 10^5 / 1000 = 200 \text{cm}^2 / \text{V} \cdot \text{s}. \]

From Figure 2-5, we find \( N_d \) is equal to \( 4.5 \times 10^{17} / \text{cm}^3 \). Hence,
\[ n = N_d = 4.5 \times 10^{17} / \text{cm}^3, \quad \text{and} \quad p = n_i^2 / n = n_i^2 / N_d = 10^{20} / 4.5 \times 10^{17} = 222 / \text{cm}^3. \]

Clearly, the minority carriers are the holes.

(b) The Fermi level with respect to \( E_c \) is
\[ E_f = E_c - kT \ln(N_d / N_c) = E_c - 0.107 \text{ eV}. \]

(c) \( R = \rho L / A. \) Using Equation (2.2.14), we first calculate the resistivity of the sample:
\[ \sigma = q \left( \mu_n n + \mu_p p \right) \approx q \mu_n n = 1.6 \times 10^{-19} \times 400 \times 4.5 \times 10^{17} = 28.8 / \Omega \cdot \text{cm}, \quad \text{and} \]
\[ \rho = \sigma^{-1} = 0.035 \Omega \cdot \text{cm}. \]
Therefore, \( R = (0.035) \times 20 \mu m / (10 \mu m \times 1.5 \mu m) = 467 \ \Omega \).

**Diffusion**

2.8 (a) Using Equation (2.3.2),
\[
J = q n \nu = q D (dn/dx).
\]
Therefore,
\[
\nu = D (1/n) (dn/dx) = -D/\lambda. \quad \text{(constant)}
\]
(b) \( J = q \mu_n n \epsilon = q n \nu \) and \( \nu = \mu_n \epsilon \).
Therefore, \( \epsilon = -D/\mu_n \lambda = -(kT/q)/\lambda \).
(c) \( \epsilon = -1000V/cm = -0.026/\lambda \). Solving for \( \lambda \) yields 0.25 \( \mu m \).

2.9 (a) \( \epsilon = -\frac{dV}{dx} = \frac{1}{q} \frac{dE_c}{dx} = \frac{1}{q L} \frac{\Delta}{L} \).
(b) \( E_c \) is parallel to \( E_v \). Hence, we can calculate the electron concentration in terms of \( E_c \).
\[
n(x) = n_0 e^{-\left(\frac{E_c(x) - E_c(0)}{kT}\right)} \quad \text{where} \quad E_c(x) - E_c(0) = \left(\frac{\Delta}{L}\right)x.
\]
Therefore, \( n(x) = n_0 e^{-x\Delta/kT} \).
(c) \( J \cdot q n \mu_n \epsilon + q D_n \frac{dn}{dx} = 0 \)
\[
q n e^{-\Delta x \mu_n/kT} \frac{\Delta}{qL} + q D_n n e^{-\Delta x \mu_n/kT} \left( -\frac{\Delta}{LkT} \right) = 0
\]
Therefore,
\[
\frac{\mu_n}{q} = \frac{D_n}{kT} \Rightarrow D_n = \frac{kT}{q} \mu_n.
\]