Problem 1 (25 pts): An N-type sample of silicon has uniform density \( (N_d = 10^{18}/\text{cm}^3) \) of arsenic, and a P-type silicon sample has a uniform density \( (N_a = 10^{16}/\text{cm}^3) \) of boron. For each sample, determine the following:

(a) The equilibrium minority-carrier concentrations at room temperature (300K). Assume full ionization of impurities.

(b) The Fermi level relative to the valence–band edge \( E_V \) in each material at 300 K.

(c) The built-in potential and the energy barrier of a PN junction based on these this N-type and P-type silicon.

Problem 2 (25 pts): Please prove the Einstein relationship between \( D \) and \( \mu \) for electron and hole:

\[
D_n = \frac{kT}{q \mu_n} \quad D_p = \frac{kT}{q \mu_p}
\]

Problem 3 (25 pts): answer the following mechanism, terms or questions precisely and briefly (feel free to use diagram, write your answer clear and clean): (1) How carrier concentration changes with temperature; (2) How to use semiconductors to build a thermoelectric generator; (3) quasi-Fermi levels: \( E_{Fn} \) and \( E_{Fp} \); (4) Avalanche breakdown for PN junction; and (5) How to use junction capacitance measurement to determine doping concentration?

Problem 4 (25 pts): For a long Si PN junction with \( N_d(x) = c x^{1/4} \) in the N side and \( N_a \) in the P side \( (c \) and \( N_a \) are constant), please find out the depletion width \( (x_n \) and \( x_p) \), electric field profile \( (E(x) \) as a function of \( x) \) and junction capacitance as a function of reverse bias \( (V) \).