ECE 645
Spring 2014

Homework 4

due Monday, March 17, 11:59PM

Problem 1

Determine how many digits are necessary to represent all possible values of the

A. sum of 256 integers, $S = X_1+X_2+\ldots+X_{256}$,
   with each X_i in the range from 0 to 511;
B. product of 256 integers, $P = Y_1 \cdot Y_2 \cdot \ldots \cdot Y_{256}$,
   with each Y_i the range from 0 to 511,

using

1. radix-2 conventional number system
2. radix-10 conventional number system.

Problem 2

Convert the following two octal (radix-8) numbers to the hexadecimal (radix-16) notation:

A. 1234567.7654321
B. 7654321.1234567

Problem 3

Represent the following two numbers

A. -121. 57421875
B. -91. 921875

using each of the listed below binary signed number representations with $k = 8$ and $l = 8$:

1. signed magnitude
2. one's complement
3. two's complement
4. biased with the base B=128
5. biased with the base B=128 – ulp.

Problem 4

Extend all numbers from Problem 3, expressed in the respective signed number representations with $k = 8$ and $l = 8$, to the numbers with the same value and the sizes of the integer and fractional part equal respectively to $k' = 12$ and $l' = 12$.

Problem 5

Determine all bits of the ANSI/IEEE standard single-precision floating-point representation of the following numbers (**Hint:** use the default rounding scheme if necessary):

A. $-12345.FEDBCA_{16} \times 4^{-71}$
B. $89.ABCDEF_{16} \times 2^{121}$
C. $-ABC.DEF12345_{16} \times 4^{-69}$
D. $-ABCD.EF12345_{16} \times 2^{112}$
E. (+infinity) - (-infinity)
F. (-infinity) / (+infinity)
G. 0/(-infinity)

Problem 6

What numbers are represented by the following hexadecimal strings, if these strings are treated as the ANSI/IEEE standard single-precision representations of real numbers. Please express the integer and fractional part of each number, as well as its exponent using decimal representation.

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A. 789A0000
B. 80420000
C. FFC00000
D. FF700000.
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Problem 7

Design a circuit implementing rounding scheme called "Round to nearest, ties to even (rtne)". Assume that the input is a signed number in the signed magnitude representation composed of 8 bits in the integer part and 4 bits in the fractional part. The output is an 8 bit integer and an overflow bit.

Draw a block diagram of this circuit using medium and low level components, such as multiplexers, half-adders, modified half-adders, full-adders, gates, etc.

Problem 8

Design a simplest possible test for the endianess of your computer (i.e., the test that determines whether the microprocessor of your computer uses a little-endian or big-endian representation of integers). Implement this test using a high-level programming language, and run it on your machine. Submit your source code, together with a short report containing the detailed info about the brand and model of the microprocessor installed in your computer, and the results of your test.

Problem 9

List at least 20 microprocessors or microcontrollers currently available on the market, and divide them into those following the little-endian, big-endian, or bi-endian convention. If a given processor is using a bi-endian convention, please investigate and describe what is a way of switching between the two supported conventions.

Problem 10

Perform the following operations in the Galois Field GF(2^5) with the multiplication defined using the following irreducible polynomial:

$P(x) = x^5 + x^2 + 1$.

A. '0C' · '1B'
B. '14' · '16'
C. '1A' · '1E'.

Bonus Problems

Problem B1

Prove the given below formula for an extension of a signed number in the one's complement representation with a *k*-bit integer part and an *l*-bit fractional part to a number with the same value, a *k'*-bit integer part, and an *l'*-bit fractional part, with $k' > k$ and $l' > l$.

$$x_{k-1}\, x_{k-1}\, x_{k-1} \,.\,.\, .x_{k-1}\, x_{k-2} \,\ldots\, x_1\, x_0 \,.\, x_{-1}\, x_{-2} \,\ldots\, x_{-l}\, x_{k-1} \,\,.\,\,.\,\,. \,.x_{k-1}$$

Problem B2

Derive the formula for an extension of a signed number in the biased representation with a *k*-bit integer part, an *l*-bit fractional part and the bias $B=2^{k-1}$ to a number with the same value, a *k'*-bit integer part, an *l'*-bit fractional part, and the bias $B'=2^{k'-1}$ with $k' > k$ and $l' > l$.

Problem B3

Consider the IEEE 32-bit standard floating-point format.

Can floating-point numbers be compared by treating their representations as signed integers in the signed magnitude representation, even when denormals and +/- infinity are considered?

If your answer is yes, please prove this property of the floating-point representation.

If your answer is no, please provide a counterexample.