Lecture 10

Basic Dividers
Required Reading

Behrooz Parhami,
Computer Arithmetic: Algorithms and Hardware Design

Chapter 13, Basic Division Schemes
13.1, Shift/Subtract Division Algorithms
13.3, Restoring Hardware Dividers
13.4, Non-Restoring and Signed Division

Chapter 15 Variation in Dividers
15.6, Combined Multiply/Divide Units
Notation and Basic Equations
## Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>Dividend</td>
<td>$z_{2k-1}z_{2k-2}\ldots z_2 z_1 z_0$</td>
</tr>
<tr>
<td>$d$</td>
<td>Divisor</td>
<td>$d_{k-1}d_{k-2}\ldots d_1 d_0$</td>
</tr>
<tr>
<td>$q$</td>
<td>Quotient</td>
<td>$q_{k-1}q_{k-2}\ldots q_1 q_0$</td>
</tr>
<tr>
<td>$s$</td>
<td>Remainder</td>
<td>$s_{k-1}s_{k-2}\ldots s_1 s_0$</td>
</tr>
<tr>
<td></td>
<td>$(s = z - dq)$</td>
<td></td>
</tr>
</tbody>
</table>
Basic Equations of Division

\[ z = dq + s \]

\[ |s| < |d| \]

\[ \text{sign}(s) = \text{sign}(z) \]

- For \( z > 0 \), \( 0 \leq s < |d| \)
- For \( z < 0 \), \(-|d| < s \leq 0\)
Unsigned Integer Division Overflow

• Must check overflow because obviously the quotient q can also be 2k bits.
  • For example, if the divisor d is 1, then the quotient q is the dividend z, which is 2k bits

Condition for no overflow (i.e. q fits in k bits):

\[ z = q \ d + s < (2^k-1) \ d + d = d \ 2^k \]

\[ z = z_H \ 2^k + z_L < d \ 2^k \]

\[ z_H < d \]
Sequential Integer Division
Basic Equations

\[ s^{(0)} = z \]

\[ s^{(j)} = 2 \ s^{(j-1)} - q_{k-j} \ (2^k \ d) \quad \text{for } j=1..k \]

\[ s^{(k)} = 2^k \ s \]
Sequential Integer Division
Justification

\[ s^{(1)} = 2z - q_{k-1} \left( 2^k d \right) \]
\[ s^{(2)} = 2(2z - q_{k-1} \left( 2^k d \right)) - q_{k-2} \left( 2^k d \right) \]
\[ s^{(3)} = 2(2(2z - q_{k-1} \left( 2^k d \right)) - q_{k-2} \left( 2^k d \right)) - q_{k-3} \left( 2^k d \right) \]

\[ \ldots \ldots \]

\[ s^{(k)} = 2(\ldots 2(2z - q_{k-1} \left( 2^k d \right)) - q_{k-2} \left( 2^k d \right)) - q_{k-3} \left( 2^k d \right) \ldots - q_0 \left( 2^k d \right) = \]
\[ = 2^k z - (2^k d) (q_{k-1} 2^{k-1} + q_{k-2} 2^{k-2} + q_{k-3} 2^{k-3} + \ldots + q_0 2^0) = \]
\[ = 2^k z - (2^k d) q = 2^k (z - d q) = 2^k s \]
**Fig. 13.2 Examples of sequential division with integer and fractional operands.**

<table>
<thead>
<tr>
<th>Integer division</th>
<th>Fractional division</th>
</tr>
</thead>
<tbody>
<tr>
<td>(z)</td>
<td>(z_{\text{frac}})</td>
</tr>
<tr>
<td>(2^4 d)</td>
<td>(d_{\text{frac}})</td>
</tr>
<tr>
<td>(s^{(0)})</td>
<td>(s^{(0)})</td>
</tr>
<tr>
<td>(2s^{(0)})</td>
<td>(2s^{(0)})</td>
</tr>
<tr>
<td>(-q_3 2^4 d)</td>
<td>(-q_{-1} d)</td>
</tr>
<tr>
<td>(s^{(1)})</td>
<td>(s^{(1)})</td>
</tr>
<tr>
<td>(2s^{(1)})</td>
<td>(2s^{(1)})</td>
</tr>
<tr>
<td>(-q_2 2^4 d)</td>
<td>(-q_{-2} d)</td>
</tr>
<tr>
<td>(s^{(2)})</td>
<td>(s^{(2)})</td>
</tr>
<tr>
<td>(2s^{(2)})</td>
<td>(2s^{(2)})</td>
</tr>
<tr>
<td>(-q_1 2^4 d)</td>
<td>(-q_{-3} d)</td>
</tr>
<tr>
<td>(s^{(3)})</td>
<td>(s^{(3)})</td>
</tr>
<tr>
<td>(2s^{(3)})</td>
<td>(2s^{(3)})</td>
</tr>
<tr>
<td>(-q_0 2^4 d)</td>
<td>(-q_{-4} d)</td>
</tr>
<tr>
<td>(s^{(4)})</td>
<td>(s^{(4)})</td>
</tr>
<tr>
<td>(s)</td>
<td>(s_{\text{frac}})</td>
</tr>
<tr>
<td>(q)</td>
<td>(q_{\text{frac}})</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
&\begin{array}{l}
z = 01110101 \\
2^4 d = 1010 \\
\end{array} \\
&\begin{array}{l}
s^{(0)} = 01110101 \\
2s^{(0)} = 01110101 \\
-q_3 2^4 d = 1010 \{q_3 = 1\} \\
\end{array} \\
&\begin{array}{l}
s^{(1)} = 0100101 \\
2s^{(1)} = 0100101 \\
-q_2 2^4 d = 0000 \{q_2 = 0\} \\
\end{array} \\
&\begin{array}{l}
s^{(2)} = 100101 \\
2s^{(2)} = 100101 \\
-q_1 2^4 d = 1010 \{q_1 = 1\} \\
\end{array} \\
&\begin{array}{l}
s^{(3)} = 10001 \\
2s^{(3)} = 10001 \\
-q_0 2^4 d = 1010 \{q_0 = 1\} \\
\end{array} \\
&\begin{array}{l}
s^{(4)} = 0111 \\
s = 0111 \\
q = 1011 \\
\end{array}
\end{align*}
\[
\begin{align*}
&\begin{array}{l}
z_{\text{frac}} = 01110101 \\
d_{\text{frac}} = 1010 \\
\end{array} \\
&\begin{array}{l}
s^{(0)} = 01110101 \\
2s^{(0)} = 0.1110101 \\
-q_{-1} d = 1.010 \{q_{-1} = 1\} \\
\end{array} \\
&\begin{array}{l}
s^{(1)} = 0100101 \\
2s^{(1)} = 0.100101 \\
-q_{-2} d = 0.000 \{q_{-2} = 0\} \\
\end{array} \\
&\begin{array}{l}
s^{(2)} = 100101 \\
2s^{(2)} = 1.00101 \\
-q_{-3} d = 1.010 \{q_{-3} = 1\} \\
\end{array} \\
&\begin{array}{l}
s^{(3)} = 10001 \\
2s^{(3)} = 1.0001 \\
-q_{-4} d = 1.010 \{q_{-4} = 1\} \\
\end{array} \\
&\begin{array}{l}
s^{(4)} = 0111 \\
s_{\text{frac}} = 0.00000111 \\
q_{\text{frac}} = 1011 \\
\end{array}
\end{align*}
\]
Fractional Division
Unsigned Fractional Division

\[ z_{\text{frac}} \text{ Dividend } \cdot Z_{-1}Z_{-2} \cdots Z_{-(2k-1)}Z_{-2k} \]

\[ d_{\text{frac}} \text{ Divisor } \cdot d_{-1}d_{-2} \cdots d_{-(k-1)}d_{-k} \]

\[ q_{\text{frac}} \text{ Quotient } \cdot q_{-1}q_{-2} \cdots q_{-(k-1)}q_{-k} \]

\[ s_{\text{frac}} \text{ Remainder } .000\ldots0s_{-(k+1)} \cdots s_{-(2k-1)}s_{-2k} \]

k bits
Integer vs. Fractional Division

For Integers:

\[ z = q \cdot 2^{-2k} \cdot d + s \cdot 2^{-2k} \]

\[ z \cdot 2^{-2k} = (q \cdot 2^{-k}) \cdot (d \cdot 2^{-k}) + s \cdot (2^{-2k}) \]

For Fractions:

\[ z_{frac} = q_{frac} \cdot d_{frac} + s_{frac} \]

where

\[ z_{frac} = z \cdot 2^{-2k} \]
\[ q_{frac} = q \cdot 2^{-k} \]
\[ d_{frac} = d \cdot 2^{-k} \]
\[ s_{frac} = s \cdot 2^{-2k} \]
Unsigned Fractional Division Overflow

Condition for no overflow:

\[ z_{\text{frac}} < d_{\text{frac}} \]
Sequential Fractional Division
Basic Equations

\[
\begin{align*}
\ s^{(0)} &= Z_{frac} \\
\ s^{(j)} &= 2 \ s^{(j-1)} - q_j \ d_{frac} \quad \text{for} \ j=1..k \\
2^k \cdot s_{frac} &= s^{(k)} \\
\ s_{frac} &= 2^{-k} \cdot s^{(k)}
\end{align*}
\]
Sequential Fractional Division

Justification

\[ s^{(1)} = 2 z_{\text{frac}} - q_{-1} d_{\text{frac}} \]
\[ s^{(2)} = 2(2 z_{\text{frac}} - q_{-1} d_{\text{frac}}) - q_{-2} d_{\text{frac}} \]
\[ s^{(3)} = 2(2(2 z_{\text{frac}} - q_{-1} d_{\text{frac}}) - q_{-2} d_{\text{frac}}) - q_{-3} d_{\text{frac}} \]

\[ \ldots \ldots \]

\[ s^{(k)} = 2(\ldots 2(2(2 z_{\text{frac}} - q_{-1} d_{\text{frac}}) - q_{-2} d_{\text{frac}}) - q_{-3} d_{\text{frac}} \ldots - q_{-k} d_{\text{frac}} = \]
\[ = 2^k z_{\text{frac}} - d_{\text{frac}} (q_{-1} 2^{k-1} + q_{-2} 2^{k-2} + q_{-3} 2^{k-3} + \ldots + q_{-k} 2^0) = \]
\[ = 2^k z_{\text{frac}} - d_{\text{frac}} 2^k (q_{-1} 2^{-1} + q_{-2} 2^{-2} + q_{-3} 2^{-3} + \ldots + q_{-k} 2^{-k}) = \]
\[ = 2^k z_{\text{frac}} - (2^k d_{\text{frac}}) q_{\text{frac}} = 2^k (z_{\text{frac}} - d_{\text{frac}} q_{\text{frac}}) = 2^k s_{\text{frac}} \]
Restoring Unsigned Integer Division
Restoring Unsigned Integer Division

\[ s^{(0)} = z \]

\textbf{for} \ j = 1 \ \textbf{to} \ k \\
\textbf{if} \ 2 \ s^{(j-1)} - 2^k \ d > 0 \\
\quad q_{k-j} = 1 \\
\quad s^{(j)} = 2 \ s^{(j-1)} - 2^k \ d \\
\textbf{else} \\
\quad q_{k-j} = 0 \\
\quad s^{(j)} = 2 \ s^{(j-1)} \\
\textbf{end for}
Fig. 13.6 Example of restoring unsigned division.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$2^4d$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-2^4d$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s^{(0)}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$2s^{(0)}$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$+(−2^4d)$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$s^{(1)}$</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$2s^{(1)}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$+(−2^4d)$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s^{(2)}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$2s^{(2)}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$+(−2^4d)$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s^{(3)}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2s^{(3)}$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$+(−2^4d)$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s^{(4)}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

No overflow, since: $(0111)_{\text{two}} < (1010)_{\text{two}}$

Positive, so set $q_3 = 1$

Negative, so set $q_2 = 0$ and restore

Positive, so set $q_1 = 1$

Positive, so set $q_0 = 1$
Fig. 13.5 Shift/subtract sequential restoring divider.

![Diagram of a shift/subtract sequential restoring divider](image)
Non-Restoring Unsigned Integer Division
Non-Restoring Unsigned Integer Division

\[ s^{(0)} = z \]
\[ s^{(1)} = 2 \cdot s^{(0)} - 2^k \cdot d \]

for \( j = 2 \) to \( k \)

if \( s^{(j-1)} \geq 0 \)

\[ q_{k-(j-1)} = 1 \]
\[ s^{(j)} = 2 \cdot s^{(j-1)} - 2^k \cdot d \]

else

\[ q_{k-(j-1)} = 0 \]
\[ s^{(j)} = 2 \cdot s^{(j-1)} + 2^k \cdot d \]

end for

if \( s^{(k)} \geq 0 \)

\[ q_0 = 1 \]

else

\[ q_0 = 0 \]

Correction step
Non-Restoring Unsigned Integer Division

Correction step

\[ s = 2^{-k} \cdot s^{(k)} \]

\[ z = q \cdot d + s \]

\[ z, q, d \geq 0 \quad s < 0 \]

\[ z = (q-1) \cdot d + (s+d) \]

\[ z = q' \cdot d + s' \]
Example of nonrestoring unsigned division

\[
\begin{array}{c|cccc|c}
& z & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\
2^4d & 0 & 1 & 0 & 1 & 0 \\
-2^4d & 1 & 0 & 1 & 1 & 0 \\
\hline
s^{(0)} & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\
2s^{(0)} & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\
+(-2^4d) & 1 & 0 & 1 & 1 & 0 \\
\hline
s^{(1)} & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
2s^{(1)} & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
+(-2^4d) & 1 & 0 & 1 & 1 & 0 \\
\hline
s^{(2)} & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
2s^{(2)} & 1 & 1 & 1 & 1 & 0 & 1 \\
+2^4d & 0 & 1 & 0 & 1 & 0 \\
\hline
s^{(3)} & 0 & 1 & 0 & 0 & 0 & 1 \\
2s^{(3)} & 1 & 0 & 0 & 0 & 1 \\
+(-2^4d) & 1 & 0 & 1 & 1 & 0 \\
\hline
s^{(4)} & 0 & 0 & 1 & 1 & 1 \\
s & 0 & 1 & 1 & 1 \\
q & 1 & 0 & 1 & 1 \\
\hline
\end{array}
\]

No overflow, since: \((0111)_{\text{two}} < (1010)_{\text{two}}\)

Positive, so subtract

Positive, so set \(q_3 = 1\) and subtract

Negative, so set \(q_2 = 0\) and add

Positive, so set \(q_1 = 1\) and subtract

Positive, so set \(q_0 = 1\)
Partial remainder variations for restoring and nonrestoring division

Example

\( (01110101)_\text{two} / (1010)_\text{two} \)

\( (117)_\text{ten} / (10)_\text{ten} \)
Non-Restoring Unsigned Integer Division

Justification

\[ s^{(j-1)} \geq 0 \]
\[ 2 \, s^{(j-1)} - 2^k \, d < 0 \]
\[ 2 \, (2 \, s^{(j-1)} ) - 2^k \, d \geq 0 \]

Restoring division

\[ s^{(j)} = 2 \, s^{(j-1)} \]
\[ s^{(j+1)} = 2 \, s^{(j)} - 2^k \, d = 4 \, s^{(j-1)} - 2^k \, d \]

Non-Restoring division

\[ s^{(j)} = 2 \, s^{(j-1)} - 2^k \, d \]
\[ s^{(j+1)} = 2 \, s^{(j)} + 2^k \, d = 2 \, (2 \, s^{(j-1)} - 2^k \, d) + 2^k \, d = 4 \, s^{(j-1)} - 2^k \, d \]
Convergence of the Partial Quotient to $q$

Example

$$(01110101)_2 / (1010)_2$$

$$(117)_{10} / (10)_{10} = (11)_{10} = (1011)_2$$

In restoring division, the partial quotient converges to $q$ from below

In nonrestoring division, the partial quotient may overshoot $q$, but converges to it after some oscillations
Signed Integer Division
Signed Integer Division

\[ |z| \quad |d| \quad \text{sign}(z) \quad \text{sign}(d) \]

\[ |q| \quad |s| \]

\[ \text{sign}(s) = \text{sign}(z) \]

\[ \text{sign}(q) = \begin{cases} + & \text{sign}(z) = \text{sign}(d) \\ - & \text{sign}(z) \neq \text{sign}(d) \end{cases} \]
Examples of Signed Integer Division

Examples of division with signed operands

\[
\begin{align*}
z &= 5 \quad d &= 3 & \Rightarrow & \quad q &= 1 & s &= 2 \\
z &= 5 \quad d &= -3 & \Rightarrow & \quad q &= -1 & s &= 2 \\
z &= -5 \quad d &= 3 & \Rightarrow & \quad q &= -1 & s &= -2 \\
z &= -5 \quad d &= -3 & \Rightarrow & \quad q &= 1 & s &= -2
\end{align*}
\]

Magnitudes of \( q \) and \( s \) are unaffected by input signs
Signs of \( q \) and \( s \) are derivable from signs of \( z \) and \( d \)
Non-Restoring Signed Integer Division
Non-Restoring Signed Integer Division

\[ s^{(0)} = z \]

for \( j = 1 \) to \( k \)

if \( \text{sign}(s^{(j-1)}) == \text{sign}(d) \)

\[ q_{k-j} = 1 \]
\[ s^{(j)} = 2 \ s^{(j-1)} - 2^k \ d = 2 \ s^{(j-1)} - q_{k-j} \ (2^k \ d) \]

else

\[ q_{k-j} = -1 \]
\[ s^{(j)} = 2 \ s^{(j-1)} + 2^k \ d = 2 \ s^{(j-1)} - q_{k-j} \ (2^k \ d) \]

end for

\[ q = \text{BSD}_2\text{'s\_comp\_conversion}(q) \]

Correction_step
Non-Restoring Signed Integer Division

Correction step

\[ s = 2^{-k} \cdot s^{(k)} \]

\[ z = q \ d + s \]

\[ \text{sign}(s) = \text{sign}(z) \]

\[ z = (q-1) \ d + (s+d) \]

\[ z = q' \ d + s' \]

\[ z = (q+1) \ d + (s-d) \]

\[ z = q'' \ d + s'' \]
Example of nonrestoring signed division

\[
\begin{array}{cccccccc}
\hline
z & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
2^4d & 1 & 1 & 0 & 0 & 1 \\
-2^4d & 0 & 0 & 1 & 1 & 1 \\
\hline
\end{array}
\]

\[
\begin{array}{cccccccc}
\hline
s^{(0)} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
2s^{(0)} & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
+2^4d & 1 & 1 & 0 & 0 & 1 \\
\hline
s^{(1)} & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\
2s^{(1)} & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\
+(-2^4d) & 0 & 0 & 1 & 1 & 1 \\
\hline
s^{(2)} & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
2s^{(2)} & 0 & 0 & 0 & 1 & 0 & 1 \\
+2^4d & 1 & 1 & 0 & 0 & 1 \\
\hline
s^{(3)} & 1 & 1 & 0 & 1 & 1 & 1 \\
2s^{(3)} & 1 & 0 & 1 & 1 & 1 \\
+(-2^4d) & 0 & 0 & 1 & 1 & 1 \\
\hline
s^{(4)} & 1 & 1 & 1 & 0 \\
+(-2^4d) & 0 & 0 & 1 & 1 & 1 \\
\hline
s^{(4)} & 0 & 0 & 1 & 0 & 1 \\
s & 0 & 1 & 0 & 1 \\
q & -1 & 1 & 1 & 1 \\
\hline
\end{array}
\]

\[
p = 0 \quad 1 \quad 0 \quad 1 \quad \text{Shift, compl MSB}
1 \quad 1 \quad 0 \quad 1 \quad 1 \quad \text{Add 1 to correct}
1 \quad 1 \quad 0 \quad 0 \quad \text{Check: } 33/(-7) = -4
\]
BSD → 2’ s Complement Conversion

\[ q = (q_{k-1} \ q_{k-2} \ldots \ q_1 \ q_0)_{\text{BSD}} = \]
\[ \overline{(p_{k-1} \ p_{k-2} \ldots \ p_1 \ p_0 \ 1)}_{2’ \text{ s complement}} \]

where

<table>
<thead>
<tr>
<th>( q_i )</th>
<th>( p_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Example:

\[ q_{\text{BSD}} = 1 \ -1 \ 1 \ 1 \]
\[ p = 1 \ 0 \ 1 \ 1 \]
\[ q_{2’ \text{ scomp}} = 0 \ 0 \ 1 \ 1 \ 1 = 0 \ 1 \ 1 \ 1 \]

no overflow if \( p_{k-2} = \overline{p_{k-1}} \) \( (q_{k-1} \neq q_{k-2}) \)
Nonrestoring Hardware Divider

- Quotient
- Partial Remainder
- Divisor
- k-bit adder
- c_{out}, c_{in}
- Complement
- MSB of $2^s(j-1)$
- q_{k-j}
- Divisor Sign
- Complement of Partial Remainder Sign
- add/sub
Multiply/Divide Unit
The control unit proceeds through necessary steps for multiplication or division (including using the appropriate shift direction).

The slight speed penalty owing to a more complex control unit is insignificant.

**Fig. 15.9** Sequential radix-2 multiply/divide unit.