Complex Number Arithmetic

Complex number in rectangular form: $z = a + jb$

Real part of the complex number: $\mathrm{Re}\,[z] = a$

Imaginary part of the complex number: $\mathrm{Im}\,[z] = b$

$j = \sqrt{-1}$ indicates which part of the complex number is the imaginary part. j itself is **not part of the imaginary part**.

Complex number in polar form: $z = |z|\,\angle z = |z|\,e^{j\angle z} = Me^{j\theta}$

Magnitude of the complex number: $|z| = M$

Phase angle of the complex number: $\angle z = \theta$

Magnitude of the complex number in terms of the real and imaginary parts:

$$M = |z| = \sqrt{a^2 + b^2}$$

Phase angle of the complex number in terms of the real and imaginary parts:

$$\theta = \angle z = \tan^{-1}\left(\frac{b}{a}\right)$$

Real part of the complex number in terms of its magnitude and phase angle:

$$a = \mathrm{Re}\,[z] = M\cos(\theta)$$

Imaginary part of the complex number in terms of its magnitude and phase angle:

$$b = \mathrm{Im}\,[z] = M\sin(\theta)$$

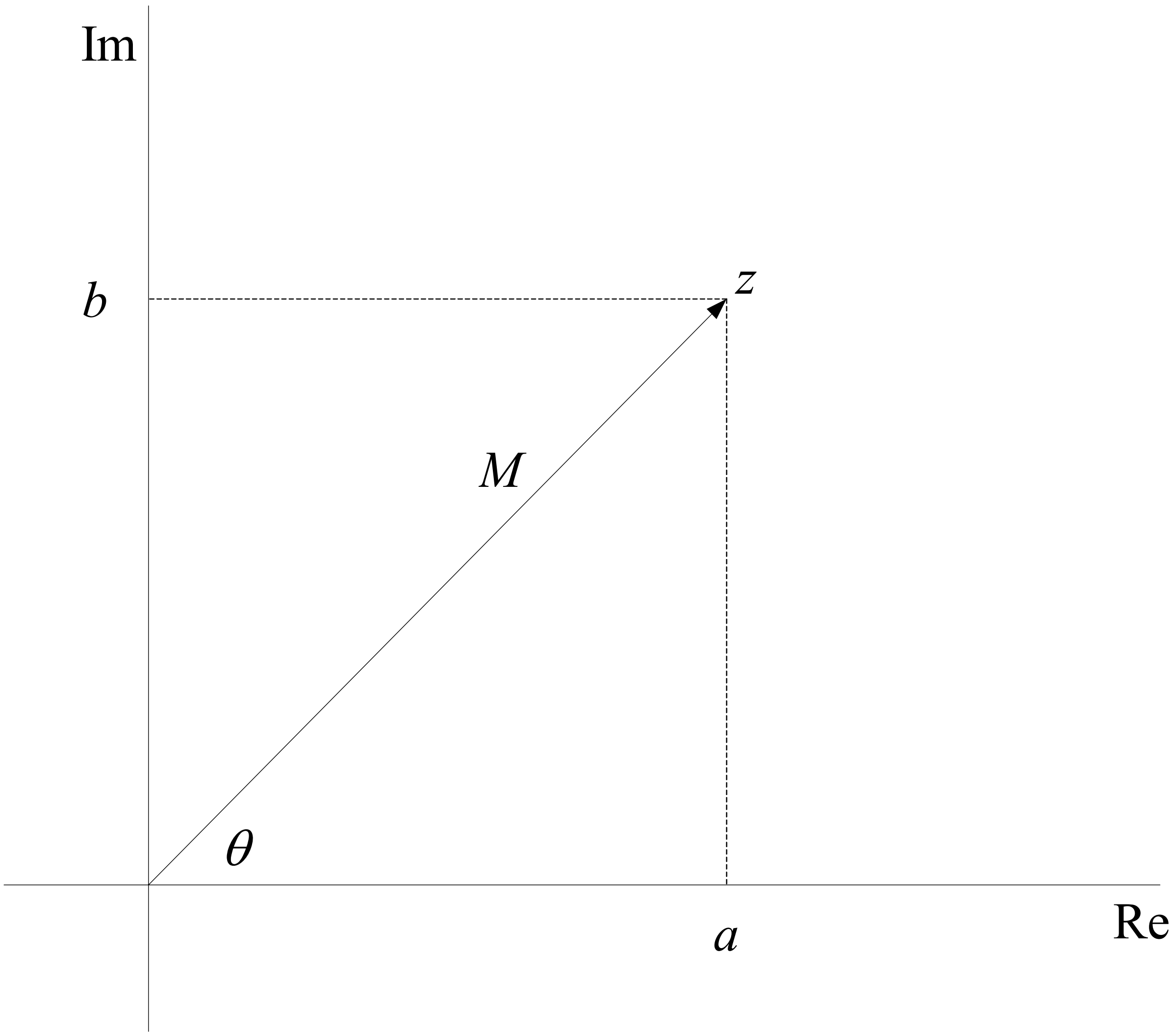
Addition and subtraction of complex numbers:

$$\begin{aligned}
z_3 &= z_1 \pm z_2 = (a_1 + jb_1) \pm (a_2 + jb_2) = (a_1 \pm a_2) + j\,(b_1 \pm b_2) \\
a_3 &= \mathrm{Re}\,[z_3] = (a_1 \pm a_2)\,, \qquad b_3 = \mathrm{Im}\,[z_3] = (b_1 \pm b_2) \\
M_3 &= |z_3| = \sqrt{(a_1 \pm a_2)^2 + (b_1 \pm b_2)^2}, \qquad \theta_3 = \angle z_3 = \tan^{-1}\left[\frac{(b_1 \pm b_2)}{(a_1 \pm a_2)}\right]
\end{aligned}$$

Multiplication and division of complex numbers:

$$\begin{aligned}
z_3 &= z_1 z_2 = \left(M_1 e^{j\theta_1}\right)\left(M_2 e^{j\theta_2}\right) = (M_1 M_2)\,e^{j(\theta_1 + \theta_2)} \\
z_3 &= z_1 z_2 = (a_1 + jb_1)\,(a_2 + jb_2) = (a_1 a_2 - b_1 b_2) + j\,(a_1 b_2 + a_2 b_1) \\
M_3 &= |z_3| = M_1 M_2 = \sqrt{a_1^2 + b_1^2} \cdot \sqrt{a_2^2 + b_2^2}, \qquad \theta_3 = \angle z_3 = \theta_1 + \theta_2 = \tan^{-1}\left(\frac{b_1}{a_1}\right) + \tan^{-1}\left(\frac{b_2}{a_2}\right) \\
a_3 &= \mathrm{Re}\,[z_3] = (a_1 a_2 - b_1 b_2)\,, \qquad b_3 = \mathrm{Im}\,[z_3] = (a_1 b_2 + a_2 b_1)
\end{aligned}$$

$$\begin{aligned}
z_3 &= \frac{z_1}{z_2} = \frac{M_1 e^{j\theta_1}}{M_2 e^{j\theta_2}} = \left(\frac{M_1}{M_2}\right) e^{j(\theta_1 - \theta_2)} \\
z_3 &= \frac{z_1}{z_2} = \frac{(a_1 + jb_1)}{(a_2 + jb_2)} = \frac{(a_1 + jb_1)\,(a_2 - jb_2)}{a_2^2 + b_2^2} = \frac{(a_1 a_2 + b_1 b_2) + j\,(a_2 b_1 - a_1 b_2)}{a_2^2 + b_2^2} \\
M_3 &= |z_3| = \frac{M_1}{M_2} = \frac{\sqrt{a_1^2 + b_1^2}}{\sqrt{a_2^2 + b_2^2}}, \qquad \theta_3 = \angle z_3 = \theta_1 - \theta_2 = \tan^{-1}\left(\frac{b_1}{a_1}\right) - \tan^{-1}\left(\frac{b_2}{a_2}\right) \\
a_3 &= \mathrm{Re}\,[z_3] = \frac{(a_1 a_2 + b_1 b_2)}{a_2^2 + b_2^2}, \qquad b_3 = \mathrm{Im}\,[z_3] = \frac{(a_2 b_1 - a_1 b_2)}{a_2^2 + b_2^2}
\end{aligned}$$



$$z = a + jb = Me^{j\theta}$$

$$M = |z| = \sqrt{a^2 + b^2}, \quad \theta = \angle z = \tan^{-1}\left(\frac{b}{a}\right)$$

$$a = \mathrm{Re}[z] = M\cos(\theta), \quad b = \mathrm{Im}[z] = M\sin(\theta)$$

A complex number shown as a vector, with its real and imaginary parts and its magnitude and phase.