George Mason University

Introduction to Signal Analysis   Spring 2017

Problem Set #3

Assigned: February 06, 2017
Due Date: February 15, 2017

Reading: Chapter 3, Sections 3.1 - 3.3 in your text, *DSP First* by McClellan, Schafer and Yoder.

Assignment: This problem set consists of two sets of problems. The first, *Practice Problems*, are *optional* and for those of you who would like some extra practice in solving problems. Answers are given to many of these problems so that you may check your work. The second set, *Regular Problems*, are the starred problems. You are to solve these problems and submit them for grading. Detailed solutions for the Regular Problems will be provided after the due date.

Important Notes:

1. Absolutely **no late homework will be accepted**.
2. It is expected that your solutions will be neat, easy to read, with clear explanations of your work and how your answers was obtained. Simply writing down an answer will not be sufficient. Part of your homework grade will be on how well it is presented.
3. You are allowed to discuss the homework problems with other students in the class. However, your homework must be written on your own, independently, and not copied in full or in part from another student. If this happens, it will be considered to be a direct violation of the George Mason honor code.

This problem set deals primarily with the spectrum of a signal, a graphical representation of the frequency content of the signal. For a single sinusoid, this spectral representation is a plot versus frequency with a line (actually a pair of lines) at the frequency of the sinusoid along with information that specifies the amplitude and phase of the sinusoid. For a sum of sinusoids, the spectrum will be a set of lines along the frequency axis, one pair for each sinusoid, with labels that indicate the amplitude and phase of each.

Key Concept

1. A sinusoid

   $$x(t) = A\cos(2\pi f_0 t + \phi)$$

   may be written in terms of complex exponentials using the inverse Euler formula

   $$\begin{aligned} x(t) &= \frac{A}{2}e^{j(2\pi f_0 t+\phi)} + \frac{A}{2}e^{-j(2\pi f_0 t+\phi)} \\ &= \frac{A}{2}e^{j\phi}e^{j2\pi f_0 t} + \frac{A}{2}e^{-j\phi}e^{-j2\pi f_0 t} \end{aligned}$$

   This represents two phasors rotating at angular frequency f_0 and $-f_0$ with (complex) amplitudes

   $$\frac{A}{2}e^{j\phi}, \qquad \frac{A}{2}e^{-j\phi}$$

   A plot of these **spectral lines** versus f is the *two-sided spectrum.*

2. A sum of sinusoids,

   $$x(t) = \sum_{k=1}^{n} A_k \cos[\omega_k t + \phi_k]$$

   will have a two-sided spectrum with **spectral lines** at frequencies $\pm f_k$ and complex amplitudes $0.5A_k e^{\pm j\phi_k}$.

Practice Problems

Problem 3.1

Simplify the following expression:

$$\left(\sqrt{3} - j\sqrt{3})\right)^{10}$$

Problem 3.2

For what value of n is the following expression equal to zero?

$$\sum_{k=0}^{n} e^{jk2\pi/6}$$

You may want to draw a picture to see what is going on.

Problem 3.3

Find all the roots of the following equation:

$$z^5 + 1 = 0$$

Problem 3.4

If the signal

$$x(t) = 2\cos(2\pi t + \pi/3)$$

is shifted in time by $t = 0.4$ seconds, how does the spectrum change?

Problem 3.5

Use the cosine indentity

$$cos^2(A) = \frac{1}{2} + \frac{1}{2}\cos(2A)$$

to plot the spectrum of the signal

$$x(t) = 4\cos^2(2\pi t + \pi/3)$$

Answers

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1. $-j6^5$
2. $n = 5$
3. The roots are

$$z = e^{j\pi(2k+1)/5}, \quad k = 0, 1, 2, 3, 4$$

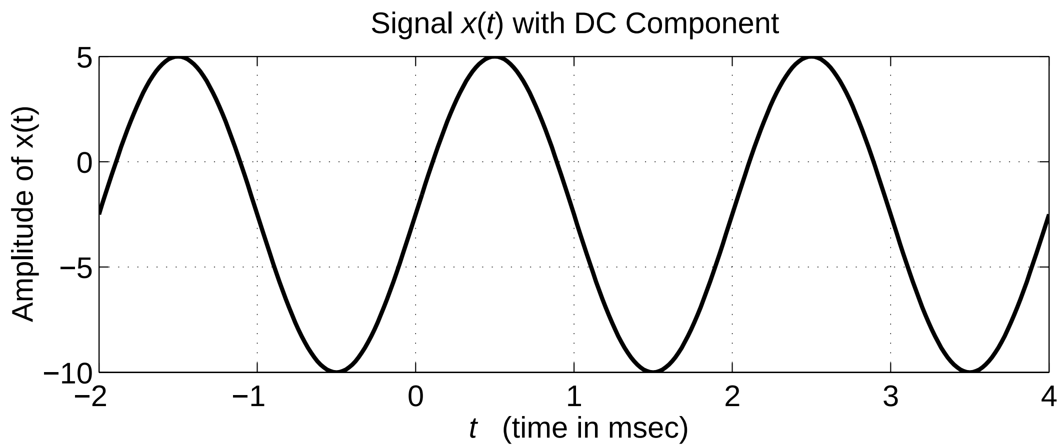
   Check your answer by computing z^5.
4. The spectral lines stay the same, but the complex amplitudes change in phase. Can you determine by how much?
5. The spectrum will have a line at $f = 0$ of amplitude 2 and spectral lines at $f = \pm 2$. What are the complex amplitudes?

Regular Problems

These problems are to be worked and submitted on blackboard by 3:00 PM on the due date.

Problem 3.1⋆

The signal $x(t)$ shown in the following plot consists of a sum of a DC component and a cosine signal. The term *DC component* means a signal that is constant versus time.



(a) The DC component may be expressed as a sinusoidal signal. What is the frequency of this sinusoid?

(b) What is the frequency of the cosine component in $x(t)$?

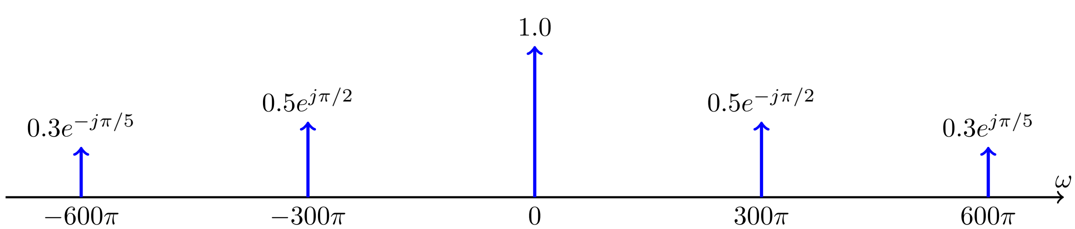
(c) Write an equation for the signal $x(t)$. You should be able to determine numerical values for all the amplitudes, frequencies, and phases in your equation by inspection of the graph above.

(d) Expand your equation in part (c) as a sum of positive and negative frequency complex exponential signals.

(e) Plot the two-sided spectrum of the signal $x(t)$. Show the complex amplitudes for each positive and negative frequency contained in $x(t)$.

Problem 3.2⋆

A real signal $x(t)$ has the following two-sided spectrum:



Note that this two-sided spectrum is plotted versus radian frequency $\omega = 2\pi f$.

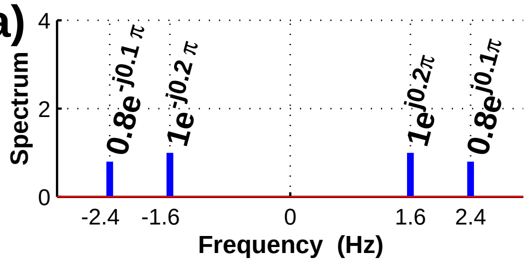
(a) Write an equation for $x(t)$ as a sum of cosines.

(b) Plot the spectrum of the signal $y(t) = 2x(t) + 10\cos(250\pi(t - 0.002))$.

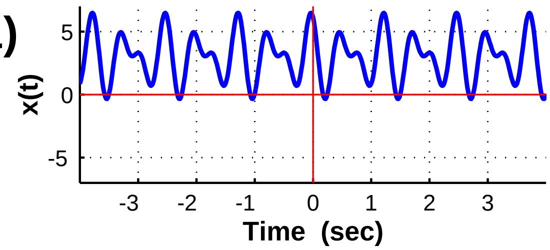
Problem 3.3★

The following plots show waveforms on the left and spectra on the right. Hand in a table matching the waveform letter with its corresponding spectrum number. Hint: You should be able to create the table just by looking at the properties of the spectrum and the properties of the signal. For example, you should be able to identify what signals (d) and (e) correspond to easily, and then by comparing (a) and (e) determine which signal (a) corresponds to.

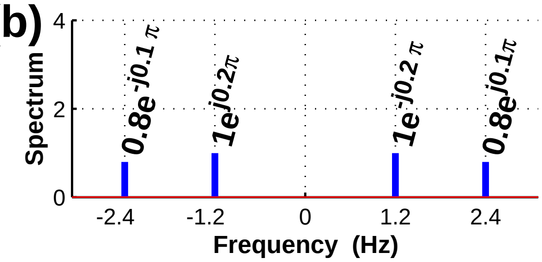
(a)



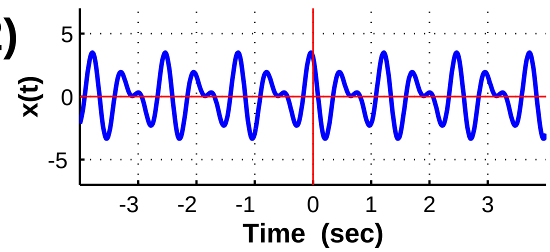
(1)



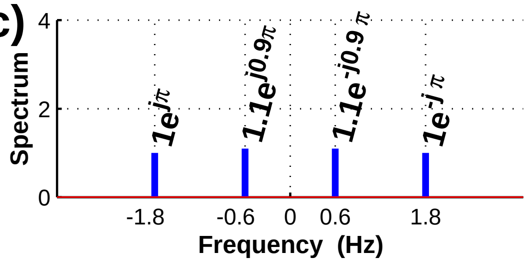
(b)



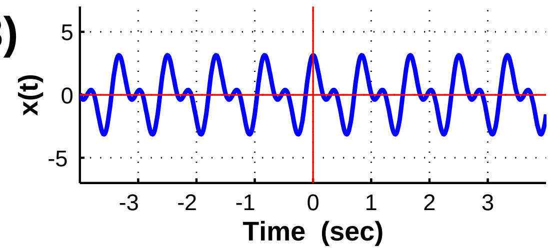
(2)



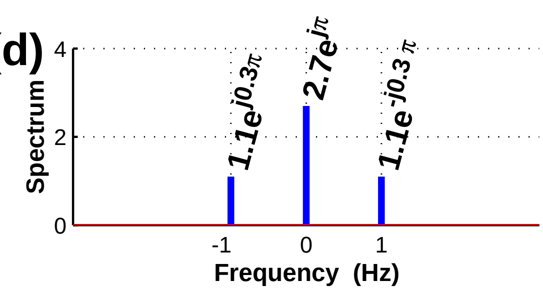
(c)



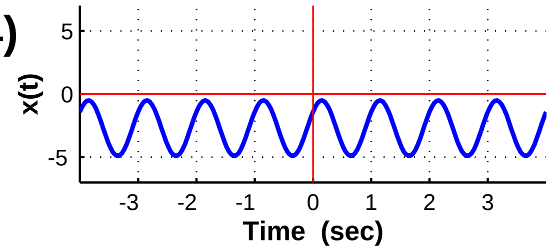
(3)



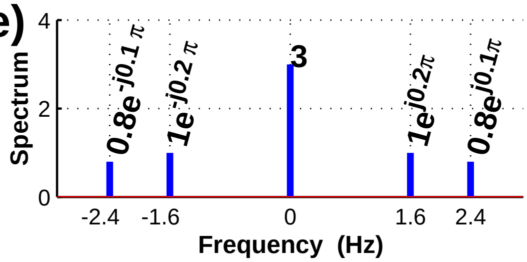
(d)



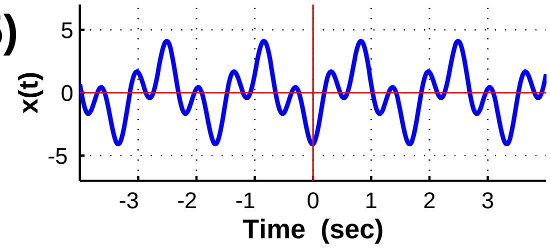
(4)



(e)



(5)



Problem 3.4★

A signal composed of sinusoids is given by the equation

$$x(t) = 2\cos(80\pi t) + 5\sin(120\pi t + \pi/4).$$

Sketch the spectrum of this signal indicating the complex amplitude of each frequency component.

Problem 3.5★

Plot the spectrum of the complex signal

$$x(t) = 3e^{j(6\pi t + 0.2\pi)} - 2e^{j(8\pi t + 0.6\pi)}$$

Problem 3.6★

Consider the signal

$$x(t) = 8[\cos(100\pi t)] \cdot [\sin(200\pi t)]$$

which is a product of two cosines having different frequencies.

(a) Using the inverse Euler relation for the sine and cosine functions, express $x(t)$ as a sum of complex exponential signals with positive and negative frequencies.

(b) Use your result in part (a) to express $x(t)$ in the form

$$x(t) = A_1\cos(\omega_0 t + \phi_1) + A_2\cos(\omega_1 t + \phi_2)$$

(c) Plot the spectrum of $x(t)$.