# George Mason University

Introduction to Signal Analysis    Spring 2017

## Problem Set #5

Assigned: February 22, 2017
Due Date: None

---

**Reading**: Chapter 4, Sections 4.1.1-4.1.5 in your text, *DSP First* by McClellan, Schafer and Yoder.

**Announcement**: The first exam will be on Monday, February 27. The exam will be closed book, closed notes, and no electronic anything will be allowed. Coverage will include all material presented in lecture through Monday, February 20, anything covered in problem sets 1-4, labs 1-4, and the reading assignments from Chapter 1-3 as well as Appendices A and B. You may also wish to read pages 102-110 in Chapter 4 since this pertains to material covered in the lecture on Februay 20.

**Assignment**: This problem set consists only of *Practice Problems*, and do not need to be turned in for grading. These are intended to give you some practice on the basics of sampling. The next problem set will also be on sampling, but will be more comprehensive.

---

**Key Concepts**

1. Sampling a continuous-time signal with a sampling frequency $f_s$, which corresponds to a sampling period of $T_s = 1/f_s$, creates a sampled signal
$$x[n] = x(nT_s)$$

2. If a sinusoidal signal of the form $x(t) = A\cos(\omega t + \phi)$ is sampled, the discrete-time signal that is formed is
$$x[n] = A\cos(\omega n T_s + \phi) = A\cos(\hat{\omega} n + \phi)$$
where
$$\boxed{\hat{\omega} \stackrel{def}{=} \omega T_s = \frac{\omega}{T_s}}$$
The signal $x[n]$ is a *sampled cosine* with a digital frequency of $\hat{\omega}$ radians.

3. Digital frequencies $\hat{\omega}$ are confined to lie between $\pm\pi$. A discrete-time sinusoid with frequency $\hat{\omega}$ is exactly the same as one with frequency $\hat{\omega} + 2\pi$ and $2\pi - \hat{\omega}$ (with a phase shift),
$$\boxed{x[n] = A\cos(n\hat{\omega} + \phi) = A\cos(n(\hat{\omega} + 2\pi)\phi) = A\cos(n(2\pi - \hat{\omega}) - \phi)}$$

> **The Sampling Theorem**
>
> A continuous-time signal $x(t)$ with frequencies no higher than $f_{\max}$ can be reconstructed exactly from its samples $x[n] = x(nT_s)$ if the samples are taken at a rate $T_s = 1/f_s$ with $f_s > 2f_{\max}$.

4. Twice the highest frequency of a signal, $2f_{\max}$, is called the Nyquist sampling rate. Signals that have no frequencies above some maximum $f_{\max}$ are said to be *bandlimited* signals.

## Practice Problems

### Problem 5.1

Let $x(t)$ be defined by

$$x(t) = 11\cos(7\pi t + \pi/9)$$

A discrete-time signal $x[n]$ is obtained by sampling $x(t)$ with a sampling frequency $f_s$. Therefore, the sampled signal is given by

$$x[n] = x(nT_s) = A\cos(\hat{\omega}n + \phi)$$

where $T_s = 1/f_s$. For each part below, determine the values of $A$, $\hat{\omega}$, and $\phi$, and determine whether or not the signal has been sampled above or below the Nyquist rate.

(a) $f_s = 9$

(b) $f_s = 6$

(c) $f_s = 3$.

### Problem 5.2

A discrete-time cosine

$$x[n] = 4\cos(0.1\pi n + 0.2\pi)$$

was obtained by sampling a continuous-time sinusoid,

$$x(t) = A\cos(2\pi f t + \phi)$$

with a sampling frequency of 8 kHz.

(a) Find values of $A$, $f$, and $\phi$ that will result in the given sampled signal $x[n]$.

(b) Is there more than one value of $f$ that will produce the given signal? If so, what are they? If not, why not?

### Problem 5.3

What is the minimum sampling rate that one may use for the signal

$$x(t) = 1.2\cos^2(2\pi(3500)t + 0.2\pi)$$

in order to avoid aliasing?

### Problem 5.4

Given the signal

$$x(t) = 3\cos(2000\pi t)\cos(3000\pi t)$$

(a) Draw a sketch of the spectrum of $x(t)$, making sure to carefully label the horizontal and vertical axes.

(b) What is the minimum sampling rate that may be used to sample $x(t)$ and avoid aliasing?

### Problem 5.5

What is the minimum sampling rate to avoid aliasing for the signal

$$x(t) = 2\cos(880\pi t + \pi/6)\cos(900\pi t - \pi/9)\cos(920\pi t + \pi/11)$$

**Answers**

1. For all parts, $A = 11$ and $\phi = \pi/9$. The Nyquist frequency is $\omega_c = 14\pi$, or $f_c = 7$ Hz.
   (a) $\hat{\omega} = 7\pi/9$ (above), (b) $\hat{\omega} = 7\pi/6$ (below), (c) $\hat{\omega} = 7\pi/3$ (below).
2. (a) $A = 4$, $\phi = 0.2\pi$ and $f = 400$. (b) Not unique. Frequencies of $f_k = 400 + 8000k$ for any integer $k$ will give the same sampled signal.
3. $f_s = 14$ kHz.
4. (a) The spectrum consists of lines at $f = \pm 500$ and $f = \pm 2500$ with amplitudes of $3/4$ and zero phase. (b) $f_s = 5$ kHz.
5. $f_s = 2.7$ kHz.