George Mason University

Introduction to Signal Analysis Spring 2017

Problem Set #7

Assigned: March 27, 2017
Due Date: April 05, 2017

Reading: Chapter 5, Sections 5.7-5.9 and 5.5 in your text, *DSP First* by McClellan, Schafer and Yoder.

Assignment: This problem set consists of two sets of problems. The first, *Practice Problems*, are *optional* and for those of you who would like some extra practice in solving problems. Answers are given to many of these problems so that you may check your work. The second set, *Regular Problems*, are the starred problems. You are to solve these problems and submit them for grading. Detailed solutions for the Regular Problems will be provided after the due date.

Important Notes:

1. Absolutely **no late homework will be accepted**.
2. It is expected that your solutions will be neat, easy to read, with clear explanations of your work and how your answers were obtained. Simply writing down an answer will not be sufficient. Part of your homework grade will be on how well it is presented.
3. You are allowed to discuss the homework problems with other students in the class. However, your homework must be written on your own, independently, and not copied in full or in part from another student. If this happens, it will be considered to be a direct violation of the George Mason honor code.

This problem set deals more with convolution, which is a concept that is extremely important to master and understand.

Key Concepts

1. A system that is linear and time-invariant is completely defined by its impulse response, $h[n]$, and for an input $x[n]$ the response of the system $y[n]$ is given by the *convolution sum*,

$$y[n] = \sum_{k=-\infty}^{\infty} h(k)x[n-k] = h[n] * x[n]$$

2. For an FIR filter described by the difference equation,

$$y[n] = b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M]$$

the impulse response is

$$h[n] = b_0\delta[n] + b_1\delta[n-1] + \cdots + b_M\delta[n-M] = \sum_{k=0}^{M} b_k\delta[n-k]$$

and the convolution sum is

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

3. Convolution satisfies the following properties,

$$h[n] * x[n] = x[n] * h[n] \qquad \text{Communative Property}$$
$$h[n] * (x_1[n] + x_2[n]) = h[n] * x_1[n] + h[n] * x_2[n] \qquad \text{Distributive Property}$$
$$(x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n]) \qquad \text{Associative Property}$$

4. If $x[n]$ is a finite-length sequence that is equal to zero outside the interval $[m_1, m_2]$ and if $h[n]$ is a finite-length sequence that is zero outside the interval $[n_1, n_2]$, then the convolution $y[n] = x[n] * h[n]$ will be a finite-length sequence that is equal to zero outside the interval $[\ell_1, \ell_2]$ where

$$\ell_1 = m_1 + n_1; \qquad \ell_2 = m_2 + n_2$$

Practice Problems

Problem 7.1

If $x[n] = \delta[n] + \delta[n-1]$ is the input to a linear time-invariant system with impulse response $h[n] = u[n]$, find the output of the system, $y[n] = x[n] * h[n]$.

Problem 7.2

If $x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$ is the input to a linear time-invariant system with impulse response $h[n] = \delta[n] + \delta[n-1]$, find the output of the system, $y[n] = x[n] * h[n]$.

Problem 7.3

Consider a system defined by

$$y[n] = \sum_{k=5}^{10} b_k x[n-k]$$

Note that the filter coefficients b_0, b_1, b_2, ..., b_4 are all zero. Suppose that the input $x[n]$ is non-zero only for $-5 \le n \le 15$. Show that $y[n]$ is non-zero at most over a finite interval of the form $N_1 \le n \le N_2$. Determine N_1 and N_2.

Problem 7.4

Find the convolution of two pulses of length 5, i.e.,

$$y[n] = x[n] * x[n]$$

where

$$x[n] \begin{cases} 1 & n = 0, 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

Problem 7.5

Find the convolution of the following two signals,

$$x[n] = u[n] - u[n-11]$$

$$h[n] = \sum_{k=0}^{\infty} \delta[n - 11k]$$

Answers

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1. $y[n] = u[n] + u[n-1] = \delta[n] + 2u[n-1]$.
2. $y[n] = \delta[n] + 2\delta[n-1] + 2\delta[n-2] + \delta[n-3]$.
3. $N_1 = 0$ and $N_2 = 25$.
4. $y[n] = \begin{cases} n+1 & n = 0, 1, 2, 3, 4 \\ 10-n & n = 5, 6, 7, 8, 9 \\ 0 & \text{otherwise} \end{cases}$
5. $y[n] = u[n]$.

Regular Problems

These problems are to be worked and submitted on blackboard by 3:00 PM on the due date.

Problem 7.1⋆

A second discrete-time system is defined by the input/output relation

$$y[n] = (x[n+1])^3. \tag{1}$$

(a) Determine whether or not this system is (i) linear; (ii) time-invariant.

(b) Find the output of this system, $y[n]$, when the input is

$$x[n] = 2\cos(0.6\pi n) = e^{j0.6\pi n} + e^{-j0.6\pi n}.$$

Express your answer in terms of cosine functions. Do not leave any powers of cosine functions in your answers.

(c) From part (b) we see that this system produces an output that contain frequencies that are not present in the input signal. Is it possible for a LTI system with impulse response $h[n]$ to produce an output that contains frequencies that are not present in the input signal? Explain.

Problem 7.2⋆

A linear time-invariant system is described by the difference equation

$$y[n] = \sum_{k=0}^{4}(2-k)x[n-k]$$

(a) What are the filter coefficients $\{b_k\}$ of this FIR filter.

(b) Find the impulse response, $h[n]$, for this FIR filter, and make a stem plot of $h[n]$ versus n.

(c) Find the output $y[n]$ when the input is

$$x[n] = \begin{cases} 0 & n < 0 \\ 2 & 0 \le n \le 5 \\ 0 & 5 < n \end{cases}$$

Make a plot of both $x[n]$ and $y[n]$ versus n.

Hint: This problem may be easier if you first find the response of the system to a step, $u[n]$ and then express $x[n]$ as a sum of steps. The output may then be found using the linearity property.

Problem 7.3⋆

For each of the following, find the convolution of $x[n]$ with $h[n]$,

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h(k)x[n-k]$$

(a) $x[n]$ is a unit step and $h[n] = \delta[n] - \delta[n-1]$.

(b) $x[n] = (0.2)^n \cos(0.1\pi n)$ and $h[n] = \delta[n]$.

(c) $x[n]$ and $h[n]$ are both unit step functions.

(d) $x[n] = 0.9^n u[n]$ and $h[n] = \delta[n-5]$.

(e) $x[n] = u[n] - u[n-10]$ and $h[n] = u[n] - u[n-20]$.

Problem 7.4★

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = x[n] - 2x[n-1] + x[n-2] + 3x[n-4].$$

(a) Draw a block diagram that represents this system in terms of unit-delay elements, coefficient multipliers, and adders.

(b) What is the impulse response $h[n]$ for this system? Express your answer as a sum of scaled and shifted unit impulse sequences.

(c) Use convolution to determine the output due to the input

$$x[n] = \delta[n] + \delta[n-1] - \delta[n-2]$$

Plot the output sequence $y[n]$ for $-2 \le n \le 10$.

Problem 7.5★

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

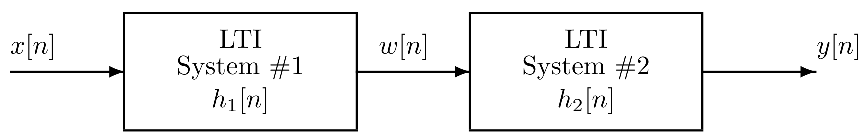

Figure 1: Cascade connection of two LTI systems.

(a) Suppose that LTI System #1 is described by the difference equation

$$w[n] = x[n] - 0.2x[n-1]$$

Determine the impulse response $h_1[n]$ of the first system.

(b) The LTI System #2 is described by the impulse response

$$h_2[n] = (0.2)^n(u[n] - u[n-L]) = \sum_{k=0}^{L-1} (0.2)^k \delta[n-k] = \begin{cases} (0.2)^n & n = 0, 1, \ldots, L-1 \\ 0 & \text{otherwise.} \end{cases}$$

For the special case of $L = 10$, use convolution to show that the impulse response of the overall cascade system is

$$h[n] = h_1[n] * h_2[n] = \delta[n] - (0.2)^{10}\delta[n-10].$$

(c) Generalize your result in part (b) for the general case of L any integer value.

(d) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1.

(e) How would you choose L so that $y[n] = x[n]$ in Figure 1; i.e., how would you choose L so that the second system "undoes" the effect of the first system?