Noise-induced bias in last principal component modeling of linear system

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Abstract

Last principal component (LPC) modeling relies on principal component transformation, and utilizes the eigenvectors associated with the last (smallest) principal components. When applied to experimental data, it may be considered an alternative to least squares based estimation of model parameters. Experimental results in the literature (cited in the body of the paper) suggest that LPC modeling is inferior to LS, in terms of estimation bias, in the presence of noise. Other results show that LPC produces unbiased estimates only in a very special case. In this paper, we derive explicit expressions for noise-induced bias in LPC-based identification. We investigate static systems with input actuator and measurement noise, and discrete dynamic systems with output measurement noise. We show that, indeed, LPC-based estimates are biased even when LS-based ones are not, and when the LS estimate is also biased, the LPC estimate has the LS bias plus an additional term. The theoretical results are supported by simulation studies.

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1. Introduction

Model-based methods constitute an important approach to fault detection and isolation (FDI). The models used for FDI are obtained from theoretical knowledge, or from empirical data. Generally model-based methods are classified into two categories: explicit model-based methods and implicit model-based methods. Diagnostic observers and parity relations belong to the former ones, those must rely on explicit models of plants or systems. Principal component analysis (PCA) is considered to belong to the latter group. It is based on an implicit model of the normal behavior, and faults can be detected by inspecting deviations from this model [3,12].

Recent research [7,9] has revealed a close link between the last principal components (LPC) of PCA transformation and parity relations. As a result, we may take advantage of analytical redundancy concepts and approaches in the construction of residuals with LPC. That extends PCA to the area of fault isolation and leads to integration with parity relations. Explicit models may also be obtained by LPC “identification”, as an alternative to the usual least squares (LS) based parameter estimation.

The PCA framework has been extended to dynamic systems [2,12,13] by including past samples of plant variables, handled as “pseudovariables”. By applying LPC modeling to the extended set of variables, one can identify the dynamic model. This modeling technique can be used also for analytical redundancy approaches in dynamic system.

In real plants, data used for identification arises from manipulated and measured variables, which are unavoidably corrupted by noise and disturbances. The traditional assumption that the input is noise free and all the noise is added to the output is not always valid. Identifying a model from noisy input–output observations is known as the errors-in-variables (EIV) problem. Using various assumptions on the noise, several approaches are presented in the literature to find accurate models under the EIV environment.

Under the assumption that both the measured input and the measured output are corrupted by white noise, the total least-square (TLS) method [21, the Frisch scheme [1], a subspace model identification (SMI)