The definition of the Chebyshev polynomials can be extended outside $|x| \leq 1$ region as follows:

$$T_m(x) = \begin{cases} \cos (m \cos^{-1}(x)) & |x| \leq 1 \\ \cosh (m \cosh^{-1}(x)) & x > 1 \\ (-1)^m \cosh(m \cosh^{-1}(x)) & x < -1 \end{cases}$$

**Useful Chebyshev Polynomial Properties:**

1. $T_m(x)$ has $m$ real roots in interval $|x| < 1$

   $$T_m(x) \bigg|_{x = \cos \left( \frac{\psi}{2} \right)} = \cos \left( m \frac{\psi}{2} \right)$$

   Roots when $\cos \left( m \frac{\psi}{2} \right) = 0$

   $$m \frac{\psi}{2} = \frac{\pi}{2} (2p - 1) \quad p = 1, \ldots, m$$

   (Non-zero multiples)

   $\Rightarrow$ Roots equally spaced in $\psi$

   $$\psi_p = \frac{\pi (2p - 1)}{m} \quad p = 1, \ldots, m$$

   In $x$-space, roots are at

   $$x_p = \cos \left( \frac{\pi (2p - 1)}{2m} \right)$$

2. $T_m(x)$ has alternating minima and maxima

   The magnitudes of these extrema are 1

   $$|T_m(x_{\text{min}})| = |T_m(x_{\text{max}})| = 1 \quad \text{for } |x| < 1$$

3. At $x = \pm 1$,

   $$|T_m(\pm 1)| = 1$$

4. For $x > 1$

   $$|T_m(x)| > 1$$