Mode Equalization at Megameter Ranges

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Abstract—Low frequency underwater sound propagation over ranges of 3.5 megameters or more has a complicated multipath arrival structure with early steep angle-arrivals, followed by an energetic finale composed of the lower order acoustic modes. Internal waves produce time-varying multipath and induce frequency-selective fading in the received signals. The low mode arrivals are strongly affected by internal waves, making it difficult to obtain precise travel time measurements for these signals. An equalizer along with suitable spatial filters, mitigates the multipath of the lower order modes. The Signal-to-Noise Ratio (SNR) measured at the output of the equalizer is used as an observable to localize modes, make time of arrival (TOA) measurements and measure the multipath spread of the modes. Results for the new mode equalizer on a simulated channel are presented. The mode equalizer is also tested on one of the North Pacific Acoustic Laboratory (NPAL) receptions.

I. INTRODUCTION

Tomographic experiments use broadband low frequency signals such as maximal length sequences (M-sequences) to study the ocean. Pulse compression (matched filtering with the transmitted sequence) provides signal-to-noise ratio (SNR) gain that is needed to observe these signals at megameter ranges. The pulse compressor output for receptions on a vertical line array (VLA) reveals the multipath structure of the signal. At long ranges in the deep ocean, the early ray arrivals are followed by an energetic finale that is best described in terms of the low order modes. The low order modes are strongly affected by scattering due to internal waves. Instead of a single strong arrival in each low mode, the signal is typically spread over 1-2 seconds.

The North Pacific Acoustic Array Laboratory (NPAL) experiment took place in 1998. A set of VLA’s off the California coast received M-sequence transmissions from a source off Kauai. One of the VLA’s was designed to be able to estimate the 10 lowest modes of the channel. Modal analysis of the NPAL receptions is quite challenging due to the high levels of noise from shipping traffic near the VLA. Bathymetric interactions near the source and receiver make the received signals more difficult to interpret. The goal of this work is to design an equalizer for the low mode signals that will make them easier to detect in this complicated high-noise environment. Having a good equalizer for the modes will facilitate time-of-arrival (TOA) and coherence measurements.

An essential problem in detecting the mode arrivals is that multipath causes time spread in the transmitted M-sequence, leading to interference between adjacent symbols.

This problem is common in wireless channels and is referred to as Inter Symbol Interference (ISI). Wireless receivers use equalizers to combat ISI. Such equalizers have a twofold advantage, namely they reduce ISI and increase SNR. Equalizers are commonly used in underwater acoustic communications (typically at ranges shorter than 3.5 Mm), e.g., [1]. Working with data from another tomography experiment, Frietag and Stojanovic demonstrated that equalization can be used for very long range underwater acoustic signals [2]. They used a Multichannel Decision Feedback Equalizer (MDFE) to process receptions from the Acoustic Thermometry of Ocean Climate Engineering Test.

While Frietag and Stojanovic’s work was motivated by a communications application, the research described below focuses on using an equalizer for tomography. This paper develops an equalizer to localize mode arrivals and provide a measurement of the multipath spread. The results are organized as follows. Sec. II uses a set of simulated receptions at megameter range to describe relevant characteristics of the underwater channel and briefly discuss conventional mode processing techniques. Sec. III develops the equalizer for the low-mode signals. Sec. IV presents results, first for a simulated data set and then for one of the NPAL receptions. Sec. V concludes the paper.

II. SIMULATED LONG-RANGE RECEPTIONS

This section describes the relevant characteristics of receptions at megameter ranges using parabolic equation (PE) simulations. The simulation environment models a 3515-km propagation path between California and Hawaii. The slowly-varying background sound speed is defined using temperature and salinity profiles from the World Ocean Atlas [3], [4]. To make the simulation realistic, the background sound speed is perturbed by 1/2 Garrett-Munk strength internal waves generated using the method of Colosi and Brown [5]. The simulation assumes that the point source is located near the sound channel axis. The 40-element vertical receiving array is similar to the one used in the NPAL experiment. It spans 300 m to 1700 m and is designed to sample the lowest 10 modes at 75 Hz.

In long-range tomography experiments such as NPAL, the source transmits phase-encoded pseudo-random signals that propagate through the channel and are recorded on a VLA. Processing of the VLA receptions includes complex
demodulation and matched filtering with the transmitted M-sequence. To simulate the pulse-compressed pressure field, the transmitted M-sequence is first convolved with the broadband channel impulse response derived from a set of PE runs. The resulting pressure signals for each hydrophone are then pulse-compressed. Fig. 1 shows the results of this processing for the long-range environment described above. For this example, the source signal is a 10-digit M-sequence BPSK-modulated on a 75 Hz carrier, and the source bandwidth is approximately 30 Hz.

Fig. 1 is representative of receptions at megameter range. The early arrivals are associated with higher order modes, which interfere constructively to form rays. The late arrivals are associated with the lowest order modes, which have the lowest group velocities in a deep water channel. Internal wave scattering has a strong effect on the low mode arrivals, resulting in a very complicated arrival pattern in the finale.

The modes are the eigenfunctions of the waveguide. Given pressure measurements made with a mode-resolving vertical array, spatial filtering can be used to estimate the signals propagating in each mode. Mode filtering algorithms assume that the pressure field can be written as a weighted sum of the known modeshapes. Since the modeshapes are frequency-dependent, the simplest way to implement the mode spatial filter is to Fourier transform the received pressure field, do narrowband processing in each frequency bin, and then inverse transform. Narrowband mode processing typically uses a least squares criterion to solve for the mode amplitudes, i.e., the estimated mode amplitudes are defined to be the product of pseudo-inverse of the sampled modeshape matrix and the measured pressure vector. For further discussion of mode filtering in the context of long-range tomography, see references [6] and [7].

Figure 2 shows the estimated signals for modes 1, 5, and 10 corresponding to the simulated pressure field in Fig. 1. As expected for a channel with internal waves, these estimates contain multiple arrivals in each mode. The multipath spread of the mode signals in the simulation is on the order of 1-2 seconds, which is similar to the time spreads observed in long-range propagation experiments [7]. Detecting highly scattered mode signals like those in Fig. 2 can be difficult, particularly when noise levels are high. (Note: the simulation does not include noise.) The next section considers the problem of equalizing the mode signals to make them easier to detect and track for tomographic applications.

### III. METHODOLOGY

The mode equalizer is inspired by the Decision Feedback Equalizer (DFE) by Peter Monsen [8]. It is equivalent to a DFE in its training mode. The DFE’s use of previous symbol decisions makes it more effective than the linear equalizer [9]. Unfortunately, the DFE is sensitive to wrong symbol decisions. In tomography the transmitted symbols are known, thus the feedback connection can be eliminated. A modified approach replaces the decisions with the transmitted symbols. Such an equalizer has all the advantages of the DFE without its high sensitivity to decision errors. Figure 3 depicts this modified DFE.

The equalizer has N2 causal taps \([c_{-1}, \ldots, c_{N2}]\) and N1 + 1 anticausal taps \([c_{0}, \ldots, c_{N1}]\) that operate on the N2 previous symbols \([s_{n}, s_{n-1}, \ldots, s_{n-N2}]\) and N1+1 samples of the received signal \([r_{n}, r_{n+1}, \ldots, r_{n+N1}]\), respectively. The causal and anticausal filters have complementary functions. The filter coefficients \([c_{-1}, \ldots, c_{N2}]\) acting on the previously
transmitted symbols cancel the ISI from the present symbol. The anticausal portion of the filter \([c_0, \cdots, c_{N2}]\) acting on the received signal serves as a time diversity combiner. It weighs different signal paths in proportion to the signal energy in each of them. It is equivalent to a modified rake.

The causal and anticausal portions of the equalizer operate in tandem to output a Minimum Mean Squared Error (MMSE) estimate \((\hat{s}_n)\) of the present symbol:

\[
\hat{s}_n = \left[ c_{N1}, \cdots, c_0, c_{-1}, \cdots, c_{-N2} \right]
\]

If \(c = \left[ c_{N1}, \cdots, c_0, c_{-1}, \cdots, c_{-N2} \right]'\) and \(u = \left[ r_{n+N1-1}, \cdots, r_0, s_{-1}, \cdots, s_{-N2} \right]'\), Eq. 1 can be written as:

\[
\hat{s}_n = c'u.
\]

The equalizer coefficients \(c\) are given by,

\[
c = R_{uu}^{-1}R_{us},
\]

where,

\[
R_{uu} = E\{uu'\}
\]

and

\[
R_{us} = E\{usn\}.
\]

The following discussion is based on [10] and quantifies the input and the output SNR needed for the tomographic application of this equalizer. For a transmitted symbol \(s_n\), the output of a multipath channel with impulse response \([h_0, h_1, \cdots, h_{N1-1}]\) and Additive White Gaussian Noise (AWGN) \(w_n\) of variance \(\sigma^2_w\) is given by:

\[
r_n = [h_0, h_1, \cdots, h_{N1-1}]\begin{bmatrix} s_n \\ s_{n+1} \\ \vdots \\ s_{N1-1} \end{bmatrix} + w_n.
\]

Using Eq. 6, the input signal to an equalizer with rake of length \(N1\) can be expressed as:

\[
\begin{bmatrix} r_n \\ r_{n+1} \\ \vdots \\ r_{N1-1} \end{bmatrix} = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{N1-1} \end{bmatrix}\begin{bmatrix} s_n \\ s_{n+1} \\ \vdots \\ s_{N1-1} \end{bmatrix} + w_n.
\]

For notational convenience, Eq. 7 can be written as:

\[
r = sng_0 + s_{n-1}g_1 + \cdots + s_{N1-1}g_{N1-1} + w,
\]

where the \(g_i\)'s are the time shifted channel responses such as,

\[
g_i = \begin{bmatrix} h_1 \\ h_{i+1} \\ \vdots \\ h_{i+N1-1} \end{bmatrix}.
\]

Note that the total signal power \(S_I\) available to the equalizer is:

\[
S_I = E_{s_n}g_0'g_0,
\]

where \(E_s = |s_n|^2\) is the power of the transmitted symbol. In Eq. 8, \(s_{n-1}g_1, \cdots, s_{N1-1}g_{N1-1}, w\) contribute to the noise. So the total noise power \(N_I\) at the input of the equalizer is,

\[
N_I = E_{s_{n-1}g_1, \cdots, s_{N1-1}g_{N1-1}} + \sigma^2_w.
\]

The input Signal to Noise Ratio (SNRI) of the equalizer is,

\[
SNRI = \frac{S_I}{N_I} = \frac{E_{s_n}g_0'g_0}{E_{s_{n-1}g_1, \cdots, s_{N1-1}g_{N1-1}} + \sigma^2_w}.
\]

The lower order modes are substantially spread and so \(E_{s_{n-1}g_1, \cdots, s_{N1-1}g_{N1-1}}\) weighs down the input SNR to the equalizer. A successful equalization scheme for the modes would decrease the ISI and exploit the time diversity to achieve an increase in SNR.

The mode equalizer based on Eqs. 3, 4 and 5 uses Eq. 2 to estimate symbols. If \(L\) is the number of equalized symbols, the Mean Squared Error (MSE) at the output of the equalizer is,

\[
MSE = \frac{1}{L}\sum_{n=1}^{N1}(s_n - \hat{s}_n)^2.
\]

The Signal to Noise Ratio at the output of the equalizer (SNRO) is:

\[
SNRO = \frac{E_b - MSE}{MSE}.
\]

The more time diversity the equalizer can exploit, the lower the MSE will be. According to Eq. 14, the MSE and the SNRO are inversely related. A decrease in MSE causes an increase in SNRO. A rake filter as long as the channel response acquires
all the time diversity of the channel and will provide a gain in SNR, i.e., \( \text{SNR}_0 \gg \text{SNR}_1 \).

The mode equalizer works as follows. First, a spatial filter is used to beamform for the desired mode and then the equalizer described above operates on the beamformer output. When the equalizer synchronizes to a time instant where it is unable to combine mode paths of high SNR, it will decode symbols with an MSE \( \approx \text{SNR}_0 \) and by Eq. 14 gives an \( \text{SNR}_0 \approx 0 \). When the equalizer synchronizes to a time instant enabling it to combine all the multipaths, it will decode symbols with a very low MSE and high \( \text{SNR}_0 \). To determine where the mode signals are arriving, the equalizer is allowed to synchronize to a series of time instants and the \( \text{SNR}_0 \) calculated from Eqs. 13 and 14. The output SNR is then used as a metric to localize the mode arrivals spread in time. The examples in the next section illustrate the operation of the mode equalizer.

IV. RESULTS

A. Results for simulated receptions

A noisy reception was simulated by adding AWGN to the simulated reception shown in Fig. 1. To get an idea of what the noisy mode signals would look like following beamforming and pulse compression, Fig. 4 shows the mode 1, 5, and 10 signals. In this case, the noise levels are low enough that the combination of the beamformer and the matched filter does pretty well. The signals in Fig. 4 look remarkably similar to the no-noise results presented in Section II.

The approach explained in Sec. III was used to process the same set of beamformed mode signals. Note that at the input to the equalizer (the output of the beamformer) the modes 1, 5 and 10 have \( \text{SNR}_1 \) equal to -9 dB, -11 dB and -14 dB respectively. These \( \text{SNR} \) calculations were based on Eq. 12. An equalizer with a rake of length 25 taps and an ISI canceler of 12 taps was used to equalize these mode signals. The rake with 25 taps, each placed at half a symbol interval (1/75 seconds) spans 1/3 second. The ISI canceler has half the number of taps, each placed at once a symbol interval (2/75 seconds).

Fig. 5 shows the performance of the equalizer for modes 1, 5 and 10. For mode 1, a 1/3-second rake synchronized to a time instant before 6 seconds is not able to combine any arrivals because mode 1 has no strong arrivals before 6.5 seconds. The equalizer following the prediction in Sec. III gives an \( \text{SNR}_0 \) of \( \approx 0 \) before 6 seconds. However, between 6.5 seconds and 7 seconds the rake combines significant arrivals in its span and achieves almost 10 dB \( \text{SNR}_0 \) for mode 1.

As Fig. 5 indicates that the equalizer does not perform as well on modes 5 and 10, as it does for mode 1, i.e., \( \text{SNR}_0 \) is comparatively low for these modes. Note that the spread of modes 5 and 10 exceeds the equalizer length. It is expected that a longer rake filter will combine more multipath arrivals and result in a higher \( \text{SNR}_0 \) for these modes. Equalizers of larger sizes were used to equalize modes 5 and 10. Fig. 6 shows the \( \text{SNR}_0 \) for a mode 5 equalizer designed with different lengths. As expected, the \( \text{SNR}_0 \) for mode 5 increases with increasing rake lengths. A 4/3 seconds equalizer gives the highest \( \text{SNR}_0 \) for mode 5.

Figure 7 shows similar curves for the mode 10 equalizer. While the 1/3 second and the 2/3 second filters do not provide any performance gain, the 1 second and 4/3 seconds filters perform better.

Fig. 8 shows how the mode 1 results depends on equalizer size. The 2/3 second equalizer performs better than the 1/3 second equalizer. While modes 5 and 10 show an increase in \( \text{SNR}_0 \) for up to equalizer lengths of 4/3 seconds, the peak \( \text{SNR}_0 \) of a 2/3, 1 and a 4/3 second equalizer are approximately equal. Mode 1 has a maximum diversity of \( \approx 1 \) second and so increasing the rake length does not yield any additional \( \text{SNR}_0 \).

For modes 1, 5 and 10 the observed spread of the \( \text{SNR}_0 \) curve (Figs. 8, 6, 7) increases with rake length. The location of the peak also changes with the length of the rake. The equalizer is able to combine arrivals over a much larger time interval, so the synchronization time vs. \( \text{SNR}_0 \) grows wider. The dependence of the width of the \( \text{SNR}_0 \) curve with respect to the rake length can be explained by equating the equalizer to an FIR filter matched to the channel response. The length of an FIR filter output depends on the length of the signal and the length of the filter response. The equalizer with an NI sample (\( \tau_{\text{rake}} \) seconds) rake has a filter response of length \( \tau_{\text{rake}} \). The input mode signal has an impulse response equal to the mode spread \( \tau_{\text{mode}} \). So the observed spread \( \tau_{\text{eq}} \) of the
SNR\textsubscript{0} curve can be written as,

\[ \tau_{\text{eq}} = \tau_{\text{rake}} + \tau_{\text{mode}} - 1. \]  

(15)

In Fig. 8 the 2/3 second long equalizer has a \( \tau_{\text{eq}} \) of 1.5 seconds. Substituting for \( \tau_{\text{eq}} = 1.5 \) seconds and \( \tau_{\text{mode}} = 2/3 \) second in Eq. 15 gives a time spread of .85 (\( \approx 1 \)) second for mode 1. This agrees with the known spread of mode 1 as indicated by Fig 1. Similar calculations for modes 5 (Fig. 6) and 10 (Fig. 6) give mode spreads of 4/3 seconds and 1 second respectively. The calculated \( \tau_{\text{mode}} \) for modes 5 and 10 also agree with Fig. 1.

A rake with length \( \tau_{\text{rake}} \) matched to the channel spread \( \tau_{\text{mode}} \) has its peak SNR\textsubscript{0} at the start of all the arrivals. The TOA of the mode can then be inferred from the peak value of the SNR\textsubscript{0} curve of the \( \tau_{\text{rake}} \) equalizer. The time spread \( \tau_{\text{mode}} \) of mode 1 was calculated to be \( \approx 1 \) second. The SNR\textsubscript{0} (Fig. 8) for a 1 second mode 1 equalizer peaks at 6.25 seconds. So the estimated TOA for mode 1 is 6.25 seconds. Similarly, modes 5 and 10 (Figs. 6 and 7, respectively) both have start times of around 5.75 seconds.

A more noisy reception was simulated by adding larger amounts of AWGN to the reception shown in Fig. 1. Fig. 9 shows the results of LS beamforming followed by pulse compression for the noisier reception. The strong mode 1 arrival is still obvious in the pulse-compressed output. Unlike
Fig. 9. Modes 1, 5 and 10 (from top to bottom respectively) for a noisier reception. Mode filtering was implemented using a narrowband pseudo-inverse spatial filter for each frequency bin.

the first example, however, the start and end times of modes 5 and 10 are beginning to be obscured by noise. The mode equalizer was designed to process such noisy mode signals. Fig. 10 shows the results of processing the noisy reception with a 4/3 seconds equalizer. Note that for this noisy example, the input SNRI's for modes 1, 5, and 10 are -18 dB, -20 dB and -23 dB, respectively. Fig. 10 demonstrates that the equalizer outputs can be useful in identifying the arrival interval for noisy mode signals. The equalizer SNRo is lower than in the first example, which is to be expected. The main advantage is that the equalizer outputs provide a clearer indication of the start and end times of the noisy mode arrivals than the pulse-compressed outputs alone.

For this equalizer to work well, it must know the amount of signal energy in each arrival. The equalizer relies on good channel estimates to calculate the relative energies of the disparate multipaths. A large mode spread will require an equalizer of comparable length. Note that an equalizer with a large number of taps needs a proportional number of samples to train its coefficients. The equalizer cannot track a fast fading channel since it will not have adequate sample support for training. The M-sequences used in these simulations are relatively long (27.28 seconds) and appear to provide the equalizer with enough samples to estimate the channel. The next section shows the results of applying the mode equalizer to an experimental data from the NPAL experiment.

B. Results with NPAL receptions

The NPAL experiment took place in 1998 [11]. Fig. 11 indicates that pulse compression yields very noisy estimates for modes 1, 5 and 10. It is difficult to identify detect any arrivals, much less determine start and end times.

Fig. 12 shows the output of the equalizer for modes 1, 5, and 10. The signals that were invisible in the pulse compressed output are detectable in the equalizer output. While the SNR is still quite low, the equalizer does show an arrival pattern for each of the modes. Mode 1 arrives between 5.75 seconds and 7 seconds, modes 5 and 10 arrive between 4.5 seconds and 6 seconds. Similar trends were observed for the other lower order modes. While more work needs to be done for the NPAL analysis, these results indicate that the mode equalizer developed in this paper will be a useful tool for obtaining information about the low mode arrivals.

V. SUMMARY

This paper developed an equalizer for the low mode signals in long-range acoustic transmissions. Results for a realistic simulated data set demonstrated the performance of the equalizer for different noise levels. The equalizer works best when the length of the rake filter is comparable to the multipath spread of the signal. Using the simulated data, Section IV showed how the equalizer could be used to produce estimates of the arrival time and spread of the lowest order modes. The main motivation for developing the equalizer is to process noisy receptions that are typical in many long-range tomography experiments. Sec. IV showed an example of applying

2The mode estimates were obtained using a sampled modeshapes beamformer, rather than the LS beamformer used previously. As discussed in [6], the sampled modeshapes (or matched filter) beamformer is less sensitive to environmental mismatch than the LS beamformer. Since the NPAL modeshapes had to be estimated using sound speed derived from a small number of measurements, some environmental mismatch is almost unavoidable. Note that mooring corrections were applied prior to beamforming.
the equalizer to noisy receptions from the NPAL experiment. While more work remains to be done for the NPAL data set, the results presented here indicate that the equalizer will be a very useful tool for detecting the low mode arrivals.

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