Exercise 6–1

**Ex: 6.1** \( i_C = I_S e^{v_{BE}/V_T} \)

\[ v_{BE1} - v_{BE1} = V_T \ln \left[ \frac{I_C}{I_C} \right] \]
\[ v_{BE2} = 700 + 25 \ln \left[ \frac{0.1}{1} \right] \]
\[ = 642 \text{ mV} \]

\[ v_{BE3} = 700 + 25 \ln \left[ \frac{10}{1} \right] \]
\[ = 758 \text{ mV} \]

**Ex: 6.2** \( \therefore \alpha = \frac{\beta}{\beta + 1} \)

\[ \frac{50}{50 + 1} < \alpha < \frac{150}{150 + 1} \]
\[ 0.980 < \alpha < 0.993 \]

**Ex: 6.3** \( I_C = I_E - I_B \)

\[ = 1.460 \text{ mA} - 0.01446 \text{ mA} \]
\[ = 1.446 \text{ mA} \]

\[ \alpha = \frac{I_C}{I_E} = \frac{1.446}{1.460} = 0.99 \]

\[ \beta = \frac{I_C}{I_B} = \frac{1.446}{0.01446} = 100 \]

\[ I_C = I_E e^{v_{BE}/V_T} \]

\[ I_S = \frac{I_C}{e^{v_{BE}/V_T}} = \frac{1.446}{e^{642/25}} \]
\[ = 1.446 e^{29.9336} \text{ mA} \]
\[ = 10^{-15} \text{ A} \]

**Ex: 6.4** \( \beta = \frac{\alpha}{1 - \alpha} \) and \( I_C = 10 \text{ mA} \)

For \( \alpha = 0.99 \), \( \beta = \frac{0.99}{1 - 0.99} = 99 \)

\[ I_B = \frac{I_C}{\beta} = \frac{10}{99} = 0.1 \text{ mA} \]

For \( \alpha = 0.98 \), \( \beta = \frac{0.98}{1 - 0.98} = 49 \)

\[ I_B = \frac{I_C}{\beta} = \frac{10}{49} = 0.2 \text{ mA} \]

**Ex: 6.5** Given:

\( I_S = 10^{-16} \text{ A} \), \( \beta = 100 \), \( I_C = 1 \text{ mA} \)

We write

\[ I_{BE} = I_{SC}/\alpha = I_S \left( 1 + \frac{1}{\beta} \right) \]
\[ = 10^{-16} \times 1.01 = 1.01 \times 10^{-16} \text{ A} \]

\( I_{SB} = \frac{I_S}{\beta} = \frac{10^{-16}}{100} = 10^{-18} \text{ A} \)

\[ V_{BE} = V_T \ln \left[ \frac{I_C}{I_S} \right] = 25 \ln \left[ \frac{1 \text{ mA}}{10^{-16}} \right] \]
\[ = 25 \times 29.9336 = 748 \text{ mV} \]

**Ex: 6.6**

\[ v_{BE} = 690 \text{ mV} \]

\( I_C = 1 \text{ mA} \)

For active range \( V_C \geq V_B \),

\[ R_{C_{\text{max}}} = \frac{V_{CC} - 0.690}{I_C} \]
\[ = \frac{5 - 0.69}{1} = 4.31 \text{ k}\Omega \]

**Ex: 6.7** \( I_S = 10^{-15} \text{ A} \)

Area \( C = 100 \times \) Area \( E \)

\( I_{SC} = 100 \times I_S = 10^{-13} \text{ A} \)

**Ex: 6.8** \( i_C = I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BC}/V_T} \)

for \( i_C = 0 \)

\[ I_S e^{v_{BE}/V_T} = I_{SC} e^{v_{BC}/V_T} \]

\[ \frac{I_{SC}}{I_S} = \frac{e^{v_{BC}/V_T}}{e^{v_{BE}/V_T}} \]
\[ = e^{(v_{BE} - v_{BC})/V_T} \]

\[ \therefore V_{CE} = V_{BE} - V_{BC} = V_T \ln \left[ \frac{I_C}{I_S} \right] \]

For collector Area = 100 \times \text{ Emitter area}

\[ V_{CE} = 25 \ln \left[ \frac{100}{1} \right] = 115 \text{ mV} \]
Exercise 6-2

Ex: 6.9 \( I_C = I_se^{v_{BE}/V_T} - I_{SC}e^{v_{BC}/V_T} \)

\[ I_B = \frac{I_S}{\beta} e^{v_{BE}/V_T} + I_{SC}e^{v_{BC}/V_T} \]

\[ \beta_{foward} = \frac{I_C}{I_B} \ll \beta \]

\[ \begin{align*}
  &\beta \frac{I_SE^{v_{BE}/V_T} - I_{SC}e^{v_{BC}/V_T}}{I_SE^{v_{BE}/V_T} + \beta I_{SC}e^{v_{BC}/V_T}} \\
  &= \beta e^{v_{BE} - v_{BC}}/V_T - I_{SC} / I_S \\
  &= \beta e^{v_{BE}/V_T} - I_{SC} / I_S \\
  &= \beta e^{v_{BE}/V_T} + \beta I_{SC} / I_S \\
  \end{align*} \]

Q.E.D.

\[ \beta_{foward} = 100 \times \frac{e^{200/25} - 100}{e^{200/25} + 100 \times 100} \]

\[ = 100 \times 0.2219 \approx 22.2 \]

Ex: 6.10

\[ I_E = \frac{I_S}{\alpha} e^{v_{BE}/V_T} \]

\[ 2 \text{ mA} = \frac{51}{50} 10^{-14} e^{v_{BE}/V_T} \]

\[ V_{BE} = 25 \ln \left[ \frac{2 \times 10^{-14} \times 50}{51} \times 10^{14} \right] \]

\[ = 650 \text{ mV} \]

\[ I_C = \frac{\beta}{\beta + 1} I_E = \frac{50}{51} \times 2 \]

\[ = 1.96 \text{ mA} \]

\[ I_B = \frac{I_C}{\beta} = \frac{1.96}{50} \Rightarrow 39.2 \mu A \]

Ex: 6.11 \( I_C = I_se^{v_{BE}/V_T} = 1.5 \text{ A} \)

\[ \therefore V_{BE} = V_T \ln \left[ 1.5 / 10^{-11} \right] \]

\[ = 25 \times 25.734 \]

\[ = 643 \text{ mV} \]

Ex: 6.12

\[ +1.5 \text{ V} \]

\[ \downarrow \quad 2 \text{ mA} \]

\[ R_C \]

\[ \quad \quad \downarrow \]

\[ V_C = 0.5 \text{ V} \]

\[ \downarrow \quad + \]

\[ V_{BE} \]

\[ \downarrow \quad - \]

\[ V_E = -V_{BE} \]

\[ I_E = \left( \frac{2}{10} \right) \text{ mA} \downarrow \]

\[ R_E \]

\[ \quad \downarrow \]

\[ -1.5 \text{ V} \]

\[ R_C = \frac{1.5 - V_C}{I_C} = \frac{1.5 - 0.5}{2} \]

\[ = 0.5 \, k\Omega = 500 \, \Omega \]

Since at \( I_C = 1 \text{ mA}, V_{BE} = 0.8 \text{ V}, \) then at \( I_C = 2 \text{ mA}, \)

\[ V_{BE} = 0.8 + 0.025 \ln \left( \frac{2}{10} \right) \]

\[ = 0.8 + 0.017 \]

\[ = 0.817 \text{ V} \]

\[ V_E = -V_{BE} = -0.817 \text{ V} \]

\[ I_E = \frac{2 \text{ mA}}{\alpha} = \frac{2}{0.99} = 2.02 \text{ mA} \]

\[ I_E = \frac{V_E - (-1.5)}{R_E} \]

Thus,

\[ R_E = \frac{-0.817 + 1.5}{2.02} = 0.338 \, k\Omega \]

\[ = 338 \, \Omega \]

Ex: 6.13

\[ +10 \text{ V} \]

\[ I_C \downarrow \]

\[ 5 \, k\Omega \]

\[ \quad \uparrow \quad I_E \]

\[ V_C \]

\[ \downarrow \quad \quad \downarrow \]

\[ V_E = 0.7 \text{ V} \]

\[ 5 \, k\Omega \]

\[ \quad \quad \downarrow \quad \downarrow \]

\[ V_E = -0.7 \text{ V} \]

\[ -10 \text{ V} \]

\[ I_E \downarrow \]

\[ 10 \, k\Omega \]
Exercise 6–3

\[ I_E = \frac{V_E - (-10)}{10} = \frac{-0.7 + 10}{10} = 0.93 \text{ mA} \]

Assuming active-mode operation,

\[ I_B = \frac{I_E}{\beta + 1} = \frac{0.93}{50 + 1} = 0.0182 \text{ mA} \]

\[ I_C = I_E - I_B = 0.93 - 0.0182 = 0.91 \text{ mA} \]

\[ V_C = 10 - I_C \times 5 = 10 - 0.91 \times 5 = 5.45 \text{ V} \]

Since \( V_C > V_B \), the transistor is operating in the active mode, as assumed.

**Ex: 6.14**

\[ V_B = 1.0 \text{ V} \]

Thus,

\[ I_B = \frac{V_B}{100 \text{ k}\Omega} = 0.01 \text{ mA} \]

\[ V_E = +1.7 \text{ V} \]

Thus,

\[ I_E = \frac{10 - V_E}{5 \text{ k}\Omega} = \frac{10 - 1.7}{5} = 1.66 \text{ mA} \]

and

\[ \beta + 1 = \frac{I_E}{I_B} = \frac{1.66}{0.01} = 166 \]

\[ \Rightarrow \beta = 165 \]

\[ \alpha = \frac{\beta}{\beta + 1} = \frac{165}{165 + 1} = 0.994 \]

Assuming active-mode operation,

\[ I_C = \alpha I_E = 0.994 \times 1.66 = 1.65 \text{ mA} \]

and

\[ V_C = -10 + 1.65 \times 5 = -1.75 \text{ V} \]

Since \( V_C < V_B \), the transistor is indeed operating in the active mode.

**Ex: 6.15**

The transistor is operating at a constant emitter current. Thus, a change in temperature of \(+30^\circ \text{C}\) results in a change in \( V_{EB} \) by

\[ \Delta V_{EB} = -2 \text{ mV} \times 30 = -60 \text{ mV} \]

Thus,

\[ \Delta V_E = -60 \text{ mV} \]

Since the collector current remains unchanged at \( aI_E \), the collector voltage does not change:

\[ \Delta V_C = 0 \text{ V} \]

**Ex: 6.16** Refer to Fig. 6.19(a):

\[ i_C = I_se^{v_{BE}/V_T} + \frac{V_{CE}}{r_o} \quad (1) \]

Now using Eqs. (6.21) and (6.22), we can express \( r_o \) as

\[ r_o = \frac{V_A}{I_se^{v_{BE}/V_T}} \]

Substituting in Eq. (1), we have

\[ i_C = I_se^{v_{BE}/V_T} \left( 1 + \frac{V_{CE}}{V_A} \right) \]

which is Eq. (6.18). Q.E.D.

**Ex: 6.17** \( r_o = \frac{V_A}{I_C} = \frac{100}{I_C} \)

At \( I_C = 0.1 \text{ mA} \), \( r_o = 1 \text{ M}\Omega \)

At \( I_C = 1 \text{ mA} \), \( r_o = 100 \text{ k}\Omega \)

At \( I_C = 10 \text{ mA} \), \( r_o = 10 \text{ k}\Omega \)
Exercise 6–4

Ex: 6.18 \[ \Delta I_C = \frac{\Delta V_{CE}}{r_n} \]
where \( r_n = \frac{V_A}{I_C} = \frac{100}{1} = 100 \, \Omega \)
\[ \Delta I_C = \frac{11 - 1}{100} = 0.1 \, mA \]
Thus, \( I_C \) becomes 1.1 mA.

Ex: 6.19

\[ V_{BB} = V_{BE} + I_B R_B \]
\[ V_{BE} = 0.7 \, V \]
\[ R_B = 10 \, k\Omega \]

(a) For operation in the active mode with \( V_{CE} = 5 \, V \),
\[ I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10 - 5}{10} = 0.5 \, mA \]
\[ I_B = \frac{I_C}{\beta} = \frac{0.5}{50} = 0.01 \, mA \]
\[ V_{BB} = V_{BE} + I_B R_B \]
\[ = 0.7 + 0.01 \times 10 = 0.8 \, V \]

(b) For operation at the edge of saturation,
\[ V_{CE} = 0.3 \, V \]
\[ I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10 - 0.3}{10} = 0.97 \, mA \]
\[ I_B = \frac{I_C}{\beta} = \frac{0.97}{50} = 0.0194 \, mA \]
\[ V_{BB} = V_{B} + I_B R_B \]
\[ = 0.7 + 0.0194 \times 10 = 0.894 \, V \]

(c) For operation deep in saturation with \( \beta_{forced} = 10 \), we have
\[ V_{CE} \approx 0.2 \, V \]
\[ I_C = \frac{10 - 0.2}{10} = 0.98 \, mA \]
\[ I_B = \frac{I_C}{\beta_{forced}} = \frac{0.98}{10} = 0.098 \, mA \]
\[ V_{BB} = V_{B} + I_B R_B \]
\[ = 0.7 + 0.098 \times 10 = 1.68 \, V \]

Ex: 6.20 For \( V_{BB} = 0 \, V \), \( I_B = 0 \) and the transistor is cut off. Thus,
\[ I_C = 0 \]
and
\[ V_C = V_{CC} = +10 \, V \]

Ex: 6.21 Refer to the circuit in Fig. 6.22 and let \( V_{BB} = 1.7 \, V \). The current \( I_B \) can be found from
\[ I_B = \frac{V_{BB} - V_B}{R_B} = \frac{1.7 - 0.7}{10} = 0.1 \, mA \]
Assuming operation in the active mode,
\[ I_C = \beta I_B = 50 \times 0.1 = 5 \, mA \]
Thus,
\[ V_C = V_{CC} - R_C I_C \]
\[ = 10 - 1 \times 5 = 5 \, V \]
which is greater than \( V_B \), verifying that the transistor is operating in the active mode, as assumed.

(a) To obtain operation at the edge of saturation, \( R_C \) must be increased to the value that results in \( V_{CE} = 0.3 \, V \):
\[ R_C = \frac{V_{CC} - 0.3}{I_C} \]
\[ = \frac{10 - 0.3}{5} = 1.94 \, k\Omega \]

(b) Further increasing \( R_C \) results in the transistor operating in saturation. To obtain saturation-mode operation with \( V_{CE} = 0.2 \, V \) and \( \beta_{forced} = 10 \), we use
\[ I_C = \beta_{forced} \times I_B \]
\[ = 10 \times 0.1 = 1 \, mA \]
The value of \( R_C \) required can be found from
\[ R_C = \frac{V_{CC} - V_{CE}}{I_C} \]
\[ = \frac{10 - 0.2}{1} = 9.8 \, k\Omega \]

Ex: 6.22 Refer to the circuit in Fig. 6.23(a) with the base voltage raised from 4 V to \( V_B \). If at this value of \( V_B \), the transistor is at the edge of saturation then,
\[ V_C = V_B - 0.4 \, V \]
Since \( I_C \propto I_E \), we can write
\[ \frac{10 - V_C}{R_C} = \frac{V_E}{R_E} = \frac{V_B - 0.7}{R_E} \]
Exercise 6-5

Thus,
\[
10 - (V_B - 0.4) = \frac{V_B - 0.7}{4.7} \times 3.3
\]
\[\Rightarrow V_B = +4.7 \, \text{V}\]

Ex: 6.23

To establish a reverse-bias voltage of 2 V across the CBJ,
\[V_C = +6 \, \text{V}\]

From the figure we see that
\[R_C = 10 - 6 = 8 \, \text{kΩ}\]
and
\[R_E = \frac{3.3}{0.5} = 6.6 \, \text{kΩ}\]
where we have assumed \(\alpha \approx 1\).

Ex: 6.24

\[I_C = 5I_B = \frac{10 - (V_B - 0.5)}{4.7}\]  
\[I_E = 6I_B = \frac{V_B - 0.7}{3.3}\]

Dividing Eq. (1) by Eq. (2), we have
\[
\frac{5}{6} = \frac{V_B - 0.7}{V_B - 0.5} \times \frac{4.7}{3.3}
\]
\[\Rightarrow V_B = +5.18 \, \text{V}\]

Ex: 6.25 Refer to the circuit in Fig. 6.26(a). The largest value for \(R_C\) while the BJT remains in the active mode corresponds to
\[V_C = +0.4 \, \text{V}\]

Since the emitter and collector currents remain unchanged, then from Fig. 6.26(b) we obtain
\[I_C = 4.6 \, \text{mA}\]

Thus,
\[R_C = \frac{V_C - (-10)}{I_C} = +0.4 + 10/4.6 = 2.26 \, \text{kΩ}\]

Ex: 6.26

For a 4-V reverse-biased voltage across the CBJ,
\[V_C = -4 \, \text{V}\]

Refer to the figure.
\[I_C = 1 \, \text{mA} = \frac{V_C - (-10)}{R_C}\]
\[\Rightarrow R_C = \frac{-4 + 10}{1} = 6 \, \text{kΩ}\]
\[R_E = \frac{10 - V_E}{I_E}\]

Assuming \(\alpha = 1\),
\[R_E = \frac{10 - 0.7}{1} = 9.3 \, \text{kΩ}\]
Exercise 6–6

**Ex: 6.27** Refer to the circuit in Fig. 6.27:

\[ I_B = \frac{5 - 0.7}{100} = 0.043 \text{ mA} \]

To ensure that the transistor remains in the active mode for \( \beta \) in the range 50 to 150, we need to select \( R_C \) so that for the highest collector current possible, the BJT reaches the edge of saturation, that is, \( V_{CE} = 0.3 \) V. Thus,

\[ V_{CE} = 0.3 = 10 - R_C I_{C_{\text{max}}} \]

where

\[ I_{C_{\text{max}}} = \beta_{\text{max}} I_B \]

\[ = 150 \times 0.043 = 6.45 \text{ mA} \]

Thus,

\[ R_C = \frac{10 - 0.3}{6.45} = 1.5 \text{ k}\Omega \]

For the lowest \( \beta \),

\[ I_C = \beta_{\text{min}} I_B \]

\[ = 50 \times 0.043 = 2.15 \text{ mA} \]

and the corresponding \( V_{CE} \) is

\[ V_{CE} = 10 - R_C I_C = 10 - 1.5 \times 2.15 \]

\[ = 6.775 \text{ V} \]

Thus, \( V_{CE} \) will range from 0.3 V to 6.8 V.

**Ex: 6.28** Refer to the solution of Example 6.10.

\[ I_E = \frac{V_{BB} - V_{BE}}{R_E + [R_B / (\beta + 1)]} \]

\[ = \frac{5 - 0.7}{3 + (33.3/51)} = 1.177 \text{ mA} \]

\[ I_C = \alpha I_E = 0.98 \times 1.177 = 1.15 \text{ mA} \]

Thus the current is reduced by

\[ \Delta I_C = 1.28 - 1.15 = 0.13 \text{ mA} \]

which is a −10% change.

**Ex: 6.29** Refer to the circuit in Fig. 6.30(b). The total current drawn from the power supply is

\[ I = 0.103 + 1.252 + 2.78 = 4.135 \text{ mA} \]

Thus, the power dissipated in the circuit is

\[ P = 15 \text{ V} \times 4.135 \text{ mA} = 62 \text{ mW} \]

**Ex: 6.30**

\[
\begin{align*}
V_{E3} &= \frac{I_{C3}}{\alpha} \times 0.47 \\
V_{C2} &= V_{E3} + 0.7 = \frac{I_{C3}}{\alpha} \times 0.47 + 0.7 \quad (1)
\end{align*}
\]

A node equation at the collector of \( Q_2 \) yields

\[ 2.75 = \frac{V_{C2}}{2.7} + \frac{I_{C3}}{\beta} \]

Substituting for \( V_{C2} \) from Eq. (1), we obtain

\[ 2.75 = \frac{(0.47 \frac{I_{C3}}{\alpha}) + 0.7}{2.7} + \frac{I_{C3}}{\beta} \]

Substituting \( \alpha = 0.99 \) and \( \beta = 100 \) and solving for \( I_{C3} \) results in

\[ I_{C3} = 13.4 \text{ mA} \]

Now, \( V_{E3} \) and \( V_{C2} \) can be determined:

\[ V_{E3} = \frac{I_{C3}}{\alpha} \times 0.47 = \frac{13.4}{0.99} \times 0.47 = +6.36 \text{ V} \]

\[ V_{C2} = V_{E3} + 0.7 = +7.06 \text{ V} \]

**Ex: 6.31**

\[ V_E = -I_E \times 1 \]

\[ V_E = 5 \text{ V} \]

\[ I_E = 1 \text{ k}\Omega \]

\[ I_E = 0 \text{ mA} \]

\[ V_E = -0 \times 1 = 0 \text{ V} \]

\[ I_E = 1 \text{ k}\Omega \]

\[ V_E = -1 \times 1 = -1 \text{ V} \]

\[ I_E = 0 \text{ mA} \]

\[ V_E = -0 \times 1 = 0 \text{ V} \]

\[ I_E = 1 \text{ k}\Omega \]

\[ V_E = -1 \times 1 = -1 \text{ V} \]

\[ I_E = 0 \text{ mA} \]

\[ V_E = -0 \times 1 = 0 \text{ V} \]

\[ I_E = 1 \text{ k}\Omega \]

\[ V_E = -1 \times 1 = -1 \text{ V} \]
From the figure we see that $Q_1$ will be off and $Q_2$ will be on. Since the base of $Q_2$ will be at a voltage higher than $-5$ V, transistor $Q_2$ will be operating in the active mode. We can write a loop equation for the loop containing the 10-kΩ resistor, the EBJ of $Q_2$ and the 1-kΩ resistor:

$$-I_E \times 1 - 0.7 - I_B \times 10 = -5$$

Substituting $I_B = I_E/(\beta + 1) = I_E/101$ and rearranging gives

$$I_E = \frac{5 - 0.7}{101} = 3.9 \text{ mA}$$

Thus,

$$V_E = -3.9 \text{ V}$$
$$V_{B2} = -4.6 \text{ V}$$
$$I_B = 0.039 \text{ mA}$$

**Ex: 6.32** With the input at $+10$ V, there is a strong possibility that the conducting transistor $Q_1$ will be saturated. Assuming this to be the case, the analysis steps will be as follows:

- $V_{CE,sat}|Q_1 = 0.2$ V
- $V_E = 5 \text{ V} - V_{CE,sat} = +4.8$ V
- $I_{E1} = \frac{4.8 \text{ V}}{1 \text{ k} \Omega} = 4.8 \text{ mA}$
- $V_{B1} = V_E + V_{BE1} = 4.8 + 0.7 = +5.5$ V
- $I_{B1} = \frac{10 - 5.5}{10} = 0.45 \text{ mA}$
- $I_{C1} = I_{E1} - I_{B1} = 4.8 - 0.45 = 4.35 \text{ mA}$
- $\beta_{forced} = \frac{I_C}{I_B} = \frac{4.35}{0.45} = 9.7$

which is lower than $\beta_{min}$, verifying that $Q_1$ is indeed saturated.

Finally, since $Q_2$ is off,

$\quad I_{C2} = 0$

**Ex: 6.33** $V_O = +10 - BV_{CEO} = 10 - 70 = -60$ V
### 6.1
1. Active
2. Saturation
3. Active
4. Saturation
5. Active
6. Cutoff

### 6.2
The EB junctions have a 4:1 area ratio.

\[ I_C = I_S e^{V_{BE}/V_T} \]

\[ 0.5 \times 10^{-3} = I_{S1} \times e^{0.75/0.025} \]

\[ \Rightarrow I_{S1} = 4.7 \times 10^{-17} \text{ A} \]

\[ I_{S2} = 4I_{S1} = 1.87 \times 10^{-16} \text{ A} \]

### 6.3
\[ I_C = I_S e^{V_{BE}/V_T} \]

\[ 200 \times 10^{-6} = I_S e^{30} \]

\[ \Rightarrow I_S = 1.87 \times 10^{-17} \text{ A} \]

For the transistor that is 32 times larger,

\[ I_S = 32 \times 1.87 \times 10^{-17} = 6 \times 10^{-16} \text{ A} \]

At \( V_{BE} = 30 \text{ V} \), the larger transistor conducts a current of

\[ I_C = 32 \times 200 \mu\text{A} = 6.4 \text{ mA} \]

At \( I_C = 1 \text{ mA} \), the base-emitter voltage of the larger transistor can be found as

\[ 1 \times 10^{-3} = 6 \times 10^{-16} e^{V_{BE}/V_T} \]

\[ V_{BE} = V_T \ln \left( \frac{1 \times 10^{-3}}{6 \times 10^{-16}} \right) = 0.704 \text{ V} \]

### 6.4
\[ \frac{I_{S1}}{I_{S2}} = \frac{A_{E1}}{A_{E2}} = \frac{200 \times 200}{0.4 \times 0.4} = 250,000 \]

\[ I_{C1} = I_{S1} e^{V_{BE1}/V_T} \]

\[ I_{C2} = I_{S2} e^{V_{BE2}/V_T} \]

For \( I_{C1} = I_{C2} \) we have

\[ e^{(V_{BE2} - V_{BE1})/V_T} = \frac{I_{C1}}{I_{C2}} = 250,000 \]

\[ V_{BE2} - V_{BE1} = 0.025 \ln(250,000) \]

\[ = 0.31 \text{ V} \]

### 6.5
\[ I_{C1} = 10^{-13} e^{700/25} = 0.145 \text{ A} = 145 \text{ mA} \]

\[ I_{C2} = 10^{-18} e^{700/25} = 1.45 \mu\text{A} \]

For the first transistor 1 to conduct a current of 1.45 \( \mu\text{A} \), its \( V_{BE} \) must be

\[ V_{BE1} = 0.025 \ln \left( \frac{1.45 \times 10^{-6}}{10^{-13}} \right) \]

\[ = 0.412 \text{ V} \]

### 6.6
Old technology:

\[ 10^{-3} = 2 \times 10^{-13} e^{V_{BE}/V_T} \]

\[ V_{BE} = 0.025 \ln \left( \frac{10^{-3}}{2 \times 10^{-13}} \right) = 0.673 \text{ V} \]

New technology:

\[ 10^{-3} = 2 \times 10^{-18} e^{V_{BE}/V_T} \]

\[ V_{BE} = 0.025 \ln \left( \frac{10^{-3}}{2 \times 10^{-18}} \right) = 0.846 \text{ V} \]

### 6.7
\[ 5 \times 10^{-3} = I_S e^{0.76/0.025} \]

\[ I_C = I_S e^{0.76/0.025} \]

Dividing Eq. (2) by Eq. (1) yields

\[ I_C = 5 \times 10^{-3} e^{-0.06/0.025} = 0.45 \text{ mA} \]

For \( I_C = 5 \mu\text{A} \),

\[ 5 \times 10^{-6} = I_S e^{V_{BE}/0.025} \]

Dividing Eq. (3) by Eq. (1) yields

\[ 10^{-3} = e^{V_{BE} - 0.76/0.025} \]

\[ V_{BE} = 0.76 + 0.025 \ln(10^{-3}) \]

\[ = 0.587 \text{ V} \]

### 6.8
\[ I_B = 10 \mu\text{A} \]

\[ I_C = 800 \mu\text{A} \]

\[ \beta = \frac{I_C}{I_B} = 80 \]

\[ \alpha = \frac{\beta}{\beta + 1} = \frac{80}{81} = 0.988 \]

### 6.9

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>0.5</th>
<th>0.8</th>
<th>0.9</th>
<th>0.95</th>
<th>0.99</th>
<th>0.995</th>
<th>0.999</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\beta}{1 - \alpha} )</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>19</td>
<td>49</td>
<td>99</td>
<td>999</td>
</tr>
</tbody>
</table>

### 6.10

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>2</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>500</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\beta}{\beta + \alpha} )</td>
<td>0.5</td>
<td>0.67</td>
<td>0.91</td>
<td>0.95</td>
<td>0.98</td>
<td>0.99</td>
<td>0.995</td>
<td>0.999</td>
</tr>
</tbody>
</table>

### 6.11
\[ \beta = \frac{\alpha}{1 - \alpha} \]

\[ \alpha \to \alpha + \Delta \alpha \]

\[ \beta \to \beta + \Delta \beta \]

\[ \beta + \Delta \beta = \frac{\alpha + \Delta \alpha}{1 - \alpha - \Delta \alpha} \]
Subtracting Eq. (1) from Eq. (2) gives
\[ \Delta \beta = \frac{\alpha + \Delta \alpha}{1 - \alpha - \Delta \alpha} - \frac{\alpha}{1 - \alpha} \]
\[ \Delta \beta = \frac{\Delta \alpha}{(1 - \alpha - \Delta \alpha)(1 - \alpha)} \] (3)
Dividing Eq. (3) by Eq. (1) gives
\[ \frac{\Delta \beta}{\beta} \approx \frac{\Delta \alpha}{\alpha} \left( 1 - \frac{\alpha - \Delta \alpha}{1 - \alpha} \right) \]
For \( \Delta \alpha \ll 1 \), the second factor on the right-hand side is approximately equal to \( \beta \). Thus
\[ \frac{\Delta \beta}{\beta} \simeq \beta \left( \frac{\Delta \alpha}{\alpha} \right) \]
Q.E.D.

For \( \Delta \beta \ll 1 \) and \( \beta = 100 \),
\[ \Delta \alpha \simeq -10\% \text{ and } \beta = 100, \]
\[ \frac{\Delta \alpha}{\alpha} \simeq -10\% = -0.1\% \]

6.12 Transistor is operating in active region:
\( \beta = 50 \rightarrow 300 \)
\( I_B = 10 \mu A \)
\( I_C = \beta I_B = 0.5 mA \rightarrow 3 mA \)
\( I_E = (\beta + 1)I_B = 0.51 mA \rightarrow 3.01 mA \)
Maximum power dissipated in transistor is
\( I_B \times 0.7 V + I_C \times V_C \)
\[ = 0.01 \times 0.7 + 3 \times 10 \simeq 30 \text{ mW} \]

6.13 \( I_C = I_S e^{\frac{V_{BE}}{V_T}} \)
\[ = 5 \times 10^{-15} e^{\frac{700}{25}} = 7.2 \text{ mA} \]
i_6 will be in the range \( \frac{7.2}{50} \text{ mA} \) to \( \frac{7.2}{200} \text{ mA} \), that is, 144 \( \mu \)A to 36 \( \mu \)A.
i_6 will be in the range \( (7.2 + 0.144) \text{ mA} \) to \( (7.2 + 0.036) \text{ mA} \), that is, 7.344 mA to 7.236 mA.

6.14 For \( i_B = 10 \mu A \),
\( i_C = i_E - i_B = 1000 - 10 = 990 \mu A \)
\[ \beta = \frac{i_C}{i_B} = \frac{990}{10} = 99 \]
\[ \alpha = \frac{\beta}{\beta + 1} = \frac{99}{100} = 0.99 \]
For \( i_B = 20 \mu A \),
\( i_C = i_E - i_B = 1000 - 20 = 980 \mu A \)
\[ \beta = \frac{i_C}{i_B} = \frac{980}{20} = 49 \]
\[ \alpha = \frac{\beta}{\beta + 1} = \frac{49}{50} = 0.98 \]
For \( i_B = 50 \mu A \),
\( i_C = i_E - i_B = 1000 - 50 = 950 \mu A \)
\[ \beta = \frac{i_C}{i_B} = \frac{950}{50} = 19 \]
\[ \alpha = \frac{\beta}{\beta + 1} = \frac{19}{20} = 0.95 \]

6.15 See Table below.

6.16 First we determine \( I_S, \beta, \) and \( \alpha \):
\[ 1 \times 10^{-3} = I_S e^{700/25} \]
\[ \Rightarrow I_S = 6.91 \times 10^{-16} \text{ A} \]
\[ \beta = \frac{I_C}{I_B} = \frac{1 \text{ mA}}{10 \mu A} = 100 \]
\[ \alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99 \]
Then we can determine \( I_{SE} \) and \( I_{SB} \):
\[ I_{SE} = \frac{I_S}{\alpha} = 6.98 \times 10^{-16} \text{ A} \]
\[ I_{SB} = \frac{I_S}{\beta} = 6.91 \times 10^{-18} \text{ A} \]
The figure on next page shows the four large-signal models, corresponding to Fig. 6.5(a) to (d), together with their parameter values.

This table belongs to Problem 6.15.

<table>
<thead>
<tr>
<th>Transistor</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{BE} ) (mV)</td>
<td>700</td>
<td>690</td>
<td>580</td>
<td>780</td>
<td>820</td>
</tr>
<tr>
<td>( I_C ) (mA)</td>
<td>1.000</td>
<td>1.000</td>
<td>0.230</td>
<td>10.10</td>
<td>73.95</td>
</tr>
<tr>
<td>( I_B ) (( \mu )A)</td>
<td>10</td>
<td>20</td>
<td>5</td>
<td>120</td>
<td>1050</td>
</tr>
<tr>
<td>( I_E ) (mA)</td>
<td>1.010</td>
<td>1.020</td>
<td>0.235</td>
<td>10.22</td>
<td>75</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.99</td>
<td>0.98</td>
<td>0.979</td>
<td>0.988</td>
<td>0.986</td>
</tr>
<tr>
<td>( \beta )</td>
<td>100</td>
<td>50</td>
<td>46</td>
<td>84</td>
<td>70</td>
</tr>
<tr>
<td>( I_S ) (A)</td>
<td>( 6.9 \times 10^{-16} )</td>
<td>( 1.0 \times 10^{-15} )</td>
<td>( 1.9 \times 10^{-14} )</td>
<td>( 2.8 \times 10^{-16} )</td>
<td>( 4.2 \times 10^{-16} )</td>
</tr>
</tbody>
</table>
The figure shows the circuit, where
\[ \alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99 \]
\[ I_{SE} = I_s / \alpha = 5 \times 10^{-15} / 0.99 = 5.05 \times 10^{-15} \text{ A} \]

The voltage at the emitter \( V_E \) is
\[ V_E = -V_{DE} = -V_T \ln \left( \frac{I_E}{I_{SE}} \right) = -0.025 \ln \left( \frac{2 \times 10^{-3}}{5.05 \times 10^{-15}} \right) = -0.668 \text{ V} \]

The voltage at the collector \( V_C \) is found from
\[ V_C = 5 - I_C \times 2 = 5 - \alpha I_E \times 2 = 5 - 0.99 \times 2 \times 2 = 1.04 \text{ V} \]

We can determine \( R_B \) from
\[ R_B = \frac{V_{CC} - V_B}{I_B} = \frac{15 - 0.633}{10^{-3}} = 1.44 \text{ M}\Omega \]
To obtain $V_{CE} = 1$ V, we select $R_c$ according to

$$R_c = \frac{V_{CC} - V_{CE}}{I_c}$$

$$= \frac{15 - 1}{0.5} = 28 \, \text{k}$$

6.19 $I_S = 10^{-15}$ A

Thus, a forward-biased EBJ conducting a current of 1 mA will have a forward voltage drop $V_{BE}$:

$$V_{BE} = V_T \ln \left( \frac{I}{I_S} \right)$$

$$= 0.025 \ln \left( \frac{10^{-3}}{10^{-15}} \right) = 0.691 \, \text{V}$$

$I_{SC} = 100I_S = 10^{-13}$ A

Thus, a forward-biased CBJ conducting a 1-mA current will have a forward voltage drop $V_{BC}$:

$$V_{BC} = V_T \ln \left( \frac{1 \times 10^{-3}}{1 \times 10^{-15}} \right) = 0.576 \, \text{V}$$

When forward-biased with 0.5 V, the emitter–base junction conducts

$$I = I_S e^{0.5/0.025}$$

$$= 10^{-15} e^{0.5/0.025} = 0.49 \, \mu\text{A}$$

and the CBJ conducts

$$I = I_{SC} e^{0.5/0.025}$$

$$= 10^{-13} e^{0.5/0.025} = 48.5 \, \mu\text{A}$$

6.20 The equations utilized are

$$v_{BC} = v_{BE} - v_{CE} = 0.7 - v_{CE}$$

$$i_{BC} = I_{SC} e^{v_{BE}/V_T} = 10^{-13} e^{v_{BE}/0.025}$$

$$i_{BE} = I_{SB} e^{v_{BC}/V_T} = 10^{-17} e^{0.7/0.025}$$

$$i_B = i_{BC} + i_{BE}$$

$$i_C = I_S e^{v_{BE}/V_T} - i_{BC} = 10^{-15} e^{0.7/0.025} - i_{BC}$$

Performing these calculations for $v_{CE} = 0.4$ V, 0.3 V, and 0.2 V, we obtain the results shown in the table below.

6.21 Dividing Eq. (6.14) by Eq. (6.15) and substituting $i_C / i_B = \beta_{\text{forced}}$ gives

$$\beta_{\text{forced}} = \frac{I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BE}/V_T}}{(I_S / \beta) e^{v_{BE}/V_T} + I_{BC} e^{v_{BE}/V_T}}$$

Dividing the numerator and denominator of the right-hand side by $I_{SC} e^{v_{BE}/V_T}$ and replacing $v_{BE} - v_{BC}$ by $V_{CE}$ at $V_{CE}$ gives

$$\beta_{\text{forced}} = \frac{1}{\beta} \left( \frac{I_S}{I_{SC}} \right) e^{V_{CE}/V_T} + 1$$

This equation can be used to obtain $e^{V_{CE}/V_T}$ and hence $V_{CE}$ as

$$\Rightarrow V_{CE} = V_T \ln \left[ \frac{I_S}{I_{SC}} \left( \frac{1 + \beta_{\text{forced}}}{1 - \beta_{\text{forced}} / \beta} \right) \right]$$

Q.E.D.

For $\beta = 100$ and $I_{SC} / I_S = 100$, we can use this equation to obtain $V_{CE}$ corresponding to the given values of $\beta_{\text{forced}}$. The results are as follows:

6.22

The emitter–base voltage $V_{EB}$ is found as the voltage drop across the diode $D_B$, whose scale

This table belongs to Problem 6.20.

<table>
<thead>
<tr>
<th>$v_{CE}$ (V)</th>
<th>$v_{BC}$ (V)</th>
<th>$i_{BC}$ (μA)</th>
<th>$i_{BE}$ (μA)</th>
<th>$i_B$ (μA)</th>
<th>$i_C$ (mA)</th>
<th>$i_C / i_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.3</td>
<td>0.016</td>
<td>14.46</td>
<td>14.48</td>
<td>1.446</td>
<td>100</td>
</tr>
<tr>
<td>0.3</td>
<td>0.4</td>
<td>0.89</td>
<td>14.46</td>
<td>15.35</td>
<td>1.445</td>
<td>94</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
<td>48.5</td>
<td>14.46</td>
<td>62.96</td>
<td>1.398</td>
<td>29</td>
</tr>
</tbody>
</table>
current is \( I_{SB} = I_S / \beta \), it is conducting a 10-\( \mu \)A current. Thus,

\[
V_{EB} = V_T \ln \left( \frac{10 \mu A}{I_{SB}} \right)
\]

where

\[
I_{SB} = \frac{I_S}{\beta} = \frac{10^{-14}}{50} = 2 \times 10^{-16} \text{ A}
\]

\[
V_{EB} = 0.025 \ln \left( \frac{10 \times 10^{-6}}{2 \times 10^{-16}} \right)
\]

= 0.616 V

Thus,

\( V_B = -V_{EB} = -0.616 \) V

The collector current can be found as

\[
I_C = \beta I_B
\]

= 50 \times 10 = 500 \mu A = 0.5 mA

The collector voltage can now be obtained from

\[
V_C = -5 + I_C \times 8.2 = -5 + 0.5 \times 8.2 = -0.9 \text{ V}
\]

The emitter current can be found as

\[
I_E = I_B + I_C = 10 + 500 = 510 \mu A
\]

= 0.51 mA

6.23 At \( i_C = 1 \) mA, \( v_{EB} = 0.7 \) V

At \( i_C = 10 \) mA,

\[
v_{EB} = 0.7 + V_T \ln \left( \frac{10}{T} \right)
\]

= 0.7 + 0.025 \ln(10) = 0.758 V

At \( i_C = 100 \) mA,

\[
v_{EB} = 0.7 + 0.025 \ln \left( \frac{100}{T} \right)
\]

= 0.815 V

Note that \( v_{EB} \) increases by about 60 mV for every decade increase in \( i_C \).

6.24

- Referring to the figure, we see that
  \[
  I_E = I_B + I_C = \frac{I_C}{\beta} + I_C
  \]

Thus,

\[
I_C = \frac{I_E}{1 + \frac{1}{\beta}} = \frac{1}{1 + \frac{1}{10}} = 0.909 \text{ mA}
\]

\( I_B = 0.091 \) mA

For direction of flow, refer to the figure.

\[
V_{EB} = V_T \ln \left( \frac{I_B}{I_{SB}} \right)
\]

where

\[
I_{SB} = \frac{I_S}{\beta} = \frac{10^{-15}}{10} = 10^{-16} \text{ A}
\]

\[
V_{EB} = 0.025 \ln \left( \frac{0.091 \times 10^{-3}}{10^{-16}} \right)
\]

= 0.688 V

Thus,

\[
V_E = V_B + V_{EB} = 0 + 0.688 = 0.688 \text{ V}
\]

If a transistor with \( \beta = 1000 \) is substituted,

\[
I_C = \frac{I_E}{1 + \frac{1}{\beta}} = \frac{1}{1 + \frac{1}{1000}} = 0.999 \text{ mA}
\]

Thus, \( I_C \) changes by 0.999 - 0.909 = 0.09 mA, a 9.9% increase.

6.25

\[
I_B = \frac{I_E}{\beta + 1} = \frac{5}{20 + 1} = 0.238 \text{ A} = 238 \text{ mA}
\]

\[
I_C = I_S e^{V_{EB}/V_T}
\]

\[
\alpha I_E = I_S e^{V_{EB}/V_T}
\]

where

\[
\alpha = \frac{20}{21} = 0.95
\]

\[
I_S = \alpha I_E e^{-V_{EB}/V_T}
\]

= 0.95 \times 5 e^{-(0.8/0.025)}

= 6 \times 10^{-14} \text{ A}

A transistor that conducts \( I_C = 1 \) mA with \( V_{EB} = 0.70 \) V has a scale current

\[
I_S = 1 \times 10^{-3} e^{-0.70/0.025} = 6.9 \times 10^{-16} \text{ A}
\]

The emitter–base junction areas of these two transistors will have the same ratio as that of their scale currents, thus

\[
\frac{\text{EBJ area of first transistor}}{\text{EBJ area of second transistor}} = \frac{6 \times 10^{-14}}{6.9 \times 10^{-16}} = 87
\]
6.26 The two missing large-signal equivalent circuits for the pnp transistor are those corresponding to the npn equivalent circuits in Fig. 6.5(b) and 6.5(d). They are shown in the figure.

6.27

6.28 (a) Refer to Fig. P6.28(a).

\[ I_1 = \frac{10.7 - 0.7}{5 \, \text{k\Omega}} = 2 \, \text{mA} \]

Assuming operation in the active mode,

\[ I_C = \alpha I_1 \simeq 2 \, \text{mA} \]

\[ V_2 = -10.7 + I_C \times 5 \]

\[ = -10.7 + 2 \times 5 = -0.7 \, \text{V} \]

Since \( V_2 \) is lower than \( V_B \), which is 0 V, the transistor is operating in the active mode, as assumed.

(b) Refer to Fig. P6.28(b).

Since \( V_C = -4 \, \text{V} \) is lower than \( V_B = -2.7 \, \text{V} \), the transistor is operating in the active mode.

\[ I_C = \frac{-4 - (-10)}{2.4 \, \text{k}\Omega} = 2.5 \, \text{mA} \]

\[ I_E = \frac{I_C}{\alpha} \simeq 2.5 \, \text{mA} \]

\[ V_3 = +12 - I_E \times 5.6 = 12 - 2.5 \times 5.6 = -2 \, \text{V} \]

(c) Refer to Fig. P6.28(c) and use

\[ I_C = \frac{0 - (-10)}{20} = 0.5 \, \text{mA} \]

Assuming active-mode operation, and utilizing the fact that \( \beta \) is large, \( I_B \simeq 0 \) and

\[ V_4 \simeq 2 \, \text{V} \]

Since \( V_C < V_B \), the transistor is indeed operating in the active region.

\[ I_S = I_E = \frac{I_C}{\alpha} \simeq 0.5 \, \text{mA} \]

(d) Refer to Fig. P6.28(d). Since the collector is connected to the base with a 10-k\( \Omega \) resistor and \( \beta \) is assumed to be very high, the voltage drop across the 10-k\( \Omega \) resistor will be close to zero and the base voltage will be equal to that of the collector:

\[ V_B = V_7 \]

This also implies active-mode operation. Now,

\[ V_E = V_B - 0.7 \]

Thus,

\[ V_E = \frac{V_E - (-10)}{3} \]

\[ = \frac{V_7 - 0.7 + 10}{3} = \frac{V_7 + 9.3}{3} \] (1)

Since \( I_B = 0 \), the collector current will be equal to the current through the 9.1-k\( \Omega \) resistor,

\[ I_C = \frac{+10 - V_7}{9.1} \] (2)

Since \( \alpha \simeq 1, I_C = I_E = I_B \) resulting in

\[ \frac{10 - V_7}{9.1} = \frac{V_7 + 9.3}{3} \]

\[ \Rightarrow V_7 = -4.5 \, \text{V} \]

and

\[ I_B = \frac{V_7 + 9.3}{3} = \frac{-4.5 + 9.3}{3} = 1.6 \, \text{mA} \]
6.29 (a) Since $V_C$ is lower than $V_B$, the transistor is operating in the active region. From the figure corresponding to Fig. P6.29(a), we see that

- $I_C = 1 \text{ mA}$
- $I_B = 0.0215 \text{ mA}$

Thus,

$$\beta \equiv \frac{I_C}{I_B} = \frac{1}{0.0215} = 46.5$$

(b) Observe that with $V_C$ at 3 V and $V_B$ at 4.3 V, the transistor is operating in the active region. Refer to the analysis shown in the figure, which leads to

$$\beta \equiv \frac{I_C}{I_B} = \frac{3.952}{0.048} = 82.3$$

(c) Observe that the transistor is operating in the active region and note the analysis performed on the circuit diagram. Thus,

$$I_C = I_E - I_B = 3 - 0.04 = 2.96 \text{ mA}$$

and

$$\beta \equiv \frac{I_C}{I_B} = \frac{2.96}{0.04} = 74$$

6.30 Since the meter resistance is small, $V_C \simeq V_B$ and the transistor is operating in the active region. To obtain $I_E = 1 \text{ mA}$, we arrange that $V_{BE} = 0.7 \text{ V}$. Since $V_C \simeq V_B$, $V_C$ must be set to 0.7 by selecting $R_C$ according to

$$V_C = 0.7 = V_{CC} - I_E R_C$$

Thus,

$$0.7 = 9 - 1 \times R_C$$

$$\Rightarrow R_C = 8.3 \text{ k}\Omega$$

Since the meter reads full scale when the current flowing through it (in this case, $I_B$ is 50 $\mu$A), a full-scale reading corresponds to

$$\beta \equiv \frac{I_C}{I_B} \simeq \frac{1 \text{ mA}}{50 \text{ $\mu$A}} = 20$$

If the meter reads 1/5 of full scale, then $I_B = 10 \text{ $\mu$A}$ and

$$\beta = \frac{1 \text{ mA}}{10 \text{ $\mu$A}} = 100$$

A meter reading of 1/10 full scale indicates that

$$\beta = \frac{1 \text{ mA}}{5 \text{ $\mu$A}} = 200$$
### 6.31

**Diagram:**

- $+2.5\ V$
- $I_C \downarrow$
- $5\ k\Omega$
- $V_C$
- $I_E$
- $10\ k\Omega$
- $V_E = -0.7\ V$
- $\beta = 50$
- $-2.5\ V$

\[
I_E = \frac{V_E - (-2.5)}{10} = \frac{-0.7 + 2.5}{10} = 0.18\ mA
\]

Assuming the transistor is operating in the active mode, we obtain

\[
I_B = \frac{I_E}{\beta + 1} = \frac{0.18}{50 + 1} = 3.5\ \mu A
\]

\[
I_C = \left(\frac{\beta}{\beta + 1}\right)I_E = \frac{50}{51} \times 0.18 = 0.176\ mA
\]

\[
V_C = +2.5 - I_C R_C = 2.5 - 0.176 \times 5 = 1.62\ V
\]

Since $V_C > V_B$, active-mode operation is verified.

\[
R_C = \frac{V_C - (-2.5)}{I_C} = \frac{-0.5 + 2.5}{0.495} = 4.04\ k\Omega \approx 4\ k\Omega
\]

The transistor $V_{BE}$ can be found from

\[
V_{BE} = 0.64 + V_T \ln\left(\frac{0.5\ mA}{0.1\ mA}\right) = 0.68\ V
\]

Thus,

\[
V_E = +0.68\ V
\]

and

\[
R_E = \frac{2.5 - 0.68}{0.5} = 3.64\ k\Omega
\]

The maximum allowable value for $R_C$ while the transistor remains in the active mode corresponds to $V_C = +0.4\ V$. Thus,

\[
R_{C\text{max}} = \frac{0.4 - (-2.5)}{0.495} = 5.86\ k\Omega
\]

### 6.32

**Diagram:**

- $+2.5\ V$
- $I_E = 0.5\ mA$
- $R_E$
- $V_E$
- $V_C = -0.5\ V$
- $I_C \downarrow$
- $R_C \uparrow$
- $-2.5\ V$

From the figure we see that $V_C = -0.5\ V$ is lower than the base voltage ($V_B = 0\ V$); thus the transistor will be operating in the active mode.

\[
I_C = \alpha I_E = \left(\frac{\beta}{\beta + 1}\right)I_E = \frac{100}{100 + 1} \times 0.5 = 0.495\ mA
\]

\[
R_E = \frac{V_E - (-1.5)}{I_E} = \frac{-0.76 + 1.5}{0.202} = 3.66\ k\Omega
\]

The required value of $R_E$ can be found from

\[
R_E = \frac{V_E - (-1.5)}{I_E} = \frac{-0.76 + 1.5}{0.202} = 3.66\ k\Omega
\]

To establish $V_C = 0.5\ V$, we select $R_C$ according to

\[
R_C = \frac{1.5 - 0.5}{0.2} = 5\ k\Omega
\]
In all circuits shown in Fig. P6.35, we assume active-mode operation and verify that this is the case at the end of the solution. The solutions are indicated on the corresponding circuit diagrams; the order of the steps is shown by the circled numbers.
6.36 \( I_{CEO} \) approximately doubles for every 10°C rise in temperature. A change in temperature from 25°C to 125°C—that is, an increase of 100°C—results in 10 doublings or, equivalently, an increase by a factor of 2\(^{10} \approx 1024\). Thus \( I_{CEO} \) becomes

\[
I_{CEO} = 10 \text{nA} \times 1024 = 10.24 \mu\text{A}
\]

6.37

From the figure we can write

\[
\begin{align*}
I_B &= \left( \frac{I_C}{\beta} \right) e^{V_{BE}/V_T} - I_{CEO} \\
I_C &= I_S e^{V_{BE}/V_T} + I_{CEO} \\
I_E &= I_S \left( 1 + \frac{1}{\beta} \right) e^{V_{BE}/V_T}
\end{align*}
\]

When the base is left open-circuited, \( i_B = 0 \) and Eq. (1) yields

\[
I_{CEO} = \left( \frac{I_C}{\beta} \right) e^{V_{BE}/V_T}
\]

or equivalently,

\[
I_S e^{V_{BE}/V_T} = \beta I_{CEO}
\]

Substituting for \( I_S e^{V_{BE}/V_T} \) in Eqs. (2) and (3) gives

\[
i_C = I_E = (\beta + 1) I_{CEO}
\]

6.38 Since the BJT is operating at a constant emitter current, its \( |V_{BE}| \) decreases by 2 mV for every \( ^{\circ}\text{C} \) rise in temperature. Thus,

\[
|V_{BE}| \text{ at } 0^\circ\text{C} = 0.7 + 0.002 \times 25 = 0.75 \text{ V} \\
|V_{BE}| \text{ at } 100^\circ\text{C} = 0.7 - 0.002 \times 75 = 0.55 \text{ V}
\]

6.39 (a) If the junction temperature rises to 50°C, which is an increase of 30°C, the EB voltage decreases to

\[
v_{BE} = 692 - 2 \times 30 = 632 \text{ mV}
\]

(b) First, we evaluate \( V_T \) at 20°C and at 50°C:

\[
V_T = \frac{kT}{q}
\]

where \( k = 8.62 \times 10^{-3} \text{ eV/K} \).

6.40 \( V_{BE} = 0.7 \text{ V} \) at \( i_C = 10 \text{ mA} \)

For \( V_{BE} = 0.5 \text{ V} \),

\[
i_C = 10 e^{(0.5 - 0.7) / 0.025} = 3.35 \mu\text{A}
\]

At a current \( i_C \) and a BE voltage \( V_{BE} \), the slope of the \( i_C-V_{BE} \) curve is \( I_S / V_T \). Thus,

\[
\text{Slope at } V_{BE} \text{ of } 700 \text{ mV} = \frac{10 \text{ mA}}{25 \text{ mV}} = 400 \text{ mA/V}
\]

\[
\text{Slope at } V_{BE} \text{ of } 500 \text{ mV} = \frac{3.35 \mu\text{A}}{25 \text{ mV}} = 0.134 \text{ mA/V}
\]

Ratio of slopes \( = \frac{400}{0.134} \approx 3000 \)

6.41 Use Eq. (6.18):

\[
i_C = I_S e^{V_{BE}/V_T} \left( 1 + \frac{V_{CE}}{V_A} \right)
\]

with \( I_S = 10^{-15} \text{ A} \) and \( V_A = 100 \text{ V} \), to get

\[
i_C = 10^{-15} e^{V_{BE}/0.025} \left( 1 + \frac{V_{CE}}{100} \right)
\]

<table>
<thead>
<tr>
<th>( V_{BE} ) (V)</th>
<th>( i_C ) (mA)</th>
<th>( i_C ) (mA)</th>
<th>( i_C ) (mA)</th>
<th>( i_C ) (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.65</td>
<td>0.196</td>
<td>1.45</td>
<td>3.21</td>
<td>4.81</td>
</tr>
<tr>
<td>0.70</td>
<td>0.196</td>
<td>1.45</td>
<td>3.21</td>
<td>4.81</td>
</tr>
<tr>
<td>0.72</td>
<td>0.196</td>
<td>1.45</td>
<td>3.21</td>
<td>4.81</td>
</tr>
<tr>
<td>0.73</td>
<td>0.196</td>
<td>1.45</td>
<td>3.21</td>
<td>4.81</td>
</tr>
<tr>
<td>0.74</td>
<td>0.196</td>
<td>1.45</td>
<td>3.21</td>
<td>4.81</td>
</tr>
</tbody>
</table>

To find the intercept of the straight-line characteristics on the \( i_C \) axis, we substitute \( V_{CE} = 0 \) and evaluate

\[
i_C = 10^{-15} e^{V_{BE}/V_T} \text{ A}
\]

for the given value of \( V_{BE} \). The slope of each straight line is equal to this value divided by 100 (\( V_A \)). Thus we obtain
<table>
<thead>
<tr>
<th>$v_{BE}$ (V)</th>
<th>0.65</th>
<th>0.70</th>
<th>0.72</th>
<th>0.73</th>
<th>0.74</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept (mA)</td>
<td>0.2</td>
<td>1.45</td>
<td>3.22</td>
<td>4.80</td>
<td>7.16</td>
</tr>
<tr>
<td>Slope (mA/V)</td>
<td>0.002</td>
<td>0.015</td>
<td>0.032</td>
<td>0.048</td>
<td>0.072</td>
</tr>
</tbody>
</table>

$$I_{c4} = 0.74\text{ V}$$  
$$v_{BE} = 0.73\text{ V}$$  
$$v_{BE} = 0.72\text{ V}$$  
$$v_{BE} = 0.70\text{ V}$$  
$$v_{BE} = 0.65\text{ V}$$

At 25°C, assume $I_E = 1$ mA. Thus,

$$V_{BE} = 0.68\text{ V}$$

$$I_1 = \frac{V_{BE}}{R_1} = \frac{0.68}{6.8 \times 10^3} = 0.1 \text{ mA}$$

$$I_E = I - I_1 = 1.1 - 0.1 = 1 \text{ mA}$$

which is the value assumed.

$$I_2 = I_1 + I_B = I_1 + \frac{I_E}{\beta + 1} = 0.1 + \frac{1}{101} = 0.11 \text{ mA}$$

Note that the currents in $R_1$ and $R_2$ differ only by the small base current, 0.01 mA. Had $I_1$ and $I_2$ been equal, then we would have had

$$I_1 R_1 = V_{BE}$$

$$I_2 R_2 \approx I_1 R_1 + V_{BE} R_2$$

$$V_E = -(I_1 R_1 + I_2 R_2)$$

$$= -V_{BE} \left( 1 + \frac{R_2}{R_1} \right)$$

$$= -V_{BE} \left( 1 + \frac{6.8}{0.68} \right) = -11 V_{BE} = -7.48 \text{ V}$$

which gives this circuit the name “$V_{BE}$ multiplier.” A more accurate value of $V_E$ can be obtained by taking $I_B$ into account:

$$V_E = -(I_1 R_1 + I_2 R_2)$$

$$= - \left( V_{BE} + \frac{R_2}{R_1} V_{BE} + I_B R_2 \right)$$

$$= - \left( 1 + \frac{R_2}{R_1} \right) V_{BE} - I_B R_2$$

$$= -7.48 - 0.01 \times 68 = -8.16 \text{ V}$$

As temperature increases, an approximate estimate for the temperature coefficient of $V_E$ can be obtained by assuming that $I_E$ remains constant and ignoring the temperature variation of $\beta$. Thus, we would be neglecting the temperature change of the $(I_B R_2)$ terms in Eq. (2). From Eq. (2) we can obtain the temperature coefficient of $V_E$ by utilizing the fact the $V_{BE}$ changes by $-2.2 \text{ mV/°C}$.

Thus,

$$\text{Temperature coefficient of } V_E$$

$$= - \left( 1 + \frac{R_2}{R_1} \right) \times -2.2$$

$$= -11 \times -2.2 = +24.2 \text{ mV/°C}$$

At 75°C, which is a temperature increase of 50°C, $V_E = -8.16 + 24.2 \times 50 = -6.95 \text{ V}$

As a check on our assumption of constant $I_E$, let us find the value of $I_E$ at 75°C:

$$I_E(75°C) = \frac{V_{BE}(75°C)}{R_1}$$

$$= \frac{0.68 - 2.2 \times 10^{-3} \times 50}{6.8}$$

$$= 0.084 \text{ mA}$$

$$I_E(75°C) = I - I_1(75°C)$$

$$= 1.1 - 0.084 = 1.016 \text{ mA}$$

which is reasonably close to the assumed value of 1 mA.

$$\text{At } I_C = 10 \text{ mA, }$$

$$\frac{I_A}{I_C} = 125 \text{ V}$$

$$125 \text{ kΩ} = \frac{V_A}{1 \text{ mA}} \Rightarrow V_A = 125 \text{ V}$$

$$r_o = 1/\text{slope}$$

$$= 1/(0.8 \times 10^{-5})$$

$$= 125 \text{ kΩ}$$

$$\frac{V_A}{I_C} = \frac{125 \text{ V}}{10 \text{ mA}} = 12.5 \text{ kΩ}$$
6.44 \( r_o = \frac{V_A}{I_C} = \frac{50 \text{ V}}{1 \text{ mA}} = 50 \text{ k} \Omega \)

Thus,

At \( I_C = 1 \text{ mA}, \quad r_o = \frac{50 \text{ V}}{1 \text{ mA}} = 50 \text{ k} \Omega \)

At \( I_C = 100 \mu \text{A}, \quad r_o = \frac{50 \text{ V}}{0.1 \text{ mA}} = 500 \text{ k} \Omega \)

6.45

\[
\begin{align*}
    i_C \text{(mA)} \\
    \hline
    -V_A & 0 & 1 & 2 & 3 & 4 \\
    0 & 1 & 1.1 & 1.2 \quad 1.1 & 1.3 \\
    v_{CE} \text{(V)} & 0 & 5 & 10 & 15 & 20
\end{align*}
\]

Slope of \( i_C-v_{CE} \) line corresponding to \( v_{BE} = 710 \text{ mV} \) is

\[
\text{Slope} = \frac{1.3 - 1.1}{15 - 5} = \frac{0.2 \text{ mA}}{10 \text{ V}} = 0.02 \text{ mA/V}
\]

Near saturation, \( V_{CE} = 0.3 \text{ V} \), thus

\[
i_C = 1.1 - 0.02 \times (5 - 0.3)
\]

\[
= 1.006 \simeq 1 \text{ mA}
\]

\( i_C \) will be 1.2 mA at,

\[
v_{CE} = 5 + \frac{1.2 - 1.1}{0.02} = 10 \text{ V}
\]

The intercept of the \( i_C-v_{CE} \) straight line on the \( i_C \) axis will be at

\[
i_C = 1.1 - 5 \times 0.02 = 1 \text{ mA}
\]

Thus, the Early voltage is obtained as

\[
\text{Slope} = \frac{i_C \text{ (at } v_{CE} = 0)}{V_A}
\]

\[
\Rightarrow V_A = \frac{1}{0.02} = 50 \text{ V}
\]

\[
r_o = \frac{V_A}{I_C} = \frac{50 \text{ V}}{1 \text{ mA}} = 50 \text{ k} \Omega
\]

which is the inverse of the slope of the \( i_C-v_{CE} \) line.

6.46 The equivalent circuits shown in the figure correspond to the circuits in Fig. 6.19.

6.47 \( \beta = \frac{i_C}{i_B} = \frac{1 \text{ mA}}{10 \mu \text{A}} = 100 \)

\[
\beta_{ac} = \frac{\Delta i_C}{\Delta i_B} = \frac{0.08 \text{ mA}}{80 \mu \text{A}} = 0.8 \text{ mA/V}
\]

\( \Delta i_C = \Delta i_B \times \beta_{ac} + \frac{\Delta v_{CE}}{r_o} \)

where

\[
r_o = \frac{V_A}{I_C} = \frac{100}{1} = 100 \text{ k} \Omega
\]

Thus,

\[
\Delta i_C = 2 \times 80 + \frac{2}{100} \times 10^3 = 180 \mu \text{A}
\]

\[
= 0.18 \text{ mA}
\]

6.48 Refer to the circuit in Fig. P6.48.

(a) For active-mode operation with \( V_C = 2 \text{ V} \):

\[
I_C = \frac{V_{CC} - V_C}{R_C} = \frac{10 - 2}{1} = 8 \text{ mA}
\]

\[
I_B = \frac{I_C}{\beta} = \frac{8}{50} = 0.16 \text{ mA}
\]

\[
V_{BB} = I_B R_B + V_{BE} = 0.16 \times 10 + 0.7 = 2.3 \text{ V}
\]

(b) For operation at the edge of saturation:

\[
V_{CE} = 0.3 \text{ V}
\]

\[
I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10 - 0.3}{1} = 9.7 \text{ mA}
\]
\[ I_B = \frac{I_C}{\beta} = \frac{9.7}{50} = 0.194 \text{ mA} \]

\[ V_{BB} = I_B R_B + V_{BE} = 0.194 \times 10 + 0.7 = 2.64 \text{ V} \]

(c) For operation deep in saturation with \( \beta_{\text{forced}} = 10 \):

\[ V_{CC} = 0.2 \text{ V} \]

\[ I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10 - 0.2}{1} = 9.8 \text{ mA} \]

\[ I_B = \frac{I_C}{\beta_{\text{forced}}} = \frac{9.8}{10} = 0.98 \text{ mA} \]

\[ V_{BB} = I_B R_B + V_{BE} = 0.98 \times 10 + 0.7 = 10.5 \text{ V} \]

6.49 Refer to the circuit in Fig. P6.48 (with \( V_{BB} = V_{CC} \)) and to the BJT equivalent circuit of Fig. 6.21:

\[ I_C = \frac{V_{CC} - 0.2}{R_C} \]

\[ I_B = \frac{V_{CC} - 0.7}{R_B} \]

\[ \beta_{\text{forced}} = \frac{I_C}{I_B} \]

Thus,

\[ \beta_{\text{forced}} = \left( \frac{V_{CC} - 0.2}{V_{CC} - 0.7} \right) \left( \frac{R_B}{R_C} \right) \]

For \( V_{CC} = 5 \text{ V} \) and \( \beta_{\text{forced}} = 10 \) and \( P_{\text{dissipated}} \leq 20 \text{ mW} \), we can proceed as follows.

Using Eq. (1) we can determine \( (R_B/R_C) \):

\[ 10 = \left( \frac{5 - 0.2}{5 - 0.7} \right) \left( \frac{R_B}{R_C} \right) \]

\[ \Rightarrow \frac{R_B}{R_C} = 8.96 \] (3)

Using Eq. (2), we can find \( I_B \):

\[ (10 + 1) \times 5 \times I_B \leq 20 \text{ mW} \]

\[ \Rightarrow I_B \leq 0.36 \text{ mA} \]

Thus,

\[ \frac{V_{CC} - 0.7}{R_B} \leq 0.36 \text{ mA} \]

\[ \Rightarrow R_B \geq 11.9 \text{ k}\Omega \]

From the table of 1% resistors in Appendix J we select

\[ R_B = 12.1 \text{ k}\Omega \]

Substituting in Eq. (3), we have

\[ R_C = 1.35 \text{ k}\Omega \]

From the table of 1% resistors in Appendix J we select

\[ R_C = 1.37 \text{ k}\Omega \]

For these values:

\[ I_C = \frac{5 - 0.2}{1.37} = 3.5 \text{ mA} \]

\[ I_B = \frac{5 - 0.7}{12.1} = 0.36 \text{ mA} \]

Thus,

\[ \beta_{\text{forced}} = \frac{3.5}{0.36} = 9.7 \]

\[ P_{\text{dissipated}} = V_{CC}(I_C + I_B) = 5 \times 3.86 = 19.2 \text{ mW} \]

6.50 Assume saturation-mode operation. From the figure we see that

\[ V_C = V_{CC} - V_{EC_{\text{sat}}} = 5 - 0.2 = 4.8 \text{ V} \]

To operate at the edge of saturation,

\[ V_{EC} = 0.3 \text{ V} \quad \text{and} \quad \frac{I_C}{I_B} = \beta = 50 \]
Thus,
\[ I_C = \frac{5 - 0.3}{1} = 4.7 \text{ mA} \]
\[ I_B = \frac{I_C}{\beta} = \frac{4.7}{80} = 0.094 \text{ mA} \]
\[ R_B = \frac{4.3}{I_B} = \frac{4.3}{0.094} = 45.7 \text{ k}\Omega \]

The analysis and the results are given on the circuit diagrams of Figs. 1 through 3. The circled numbers indicate the order of the analysis steps.

6.52

Figure 1 shows the circuit with the value of \( V_B \) that results in operation at the edge of saturation. Since \( \beta \) is very high,
\[ I_C \approx I_E \]
\[ \frac{3 - (V_B - 0.4)}{1} = \frac{V_B - 0.7}{1} \]
\[ \Rightarrow V_B = 2.05 \text{ V} \]

Figure 2 shows the circuit with the value of \( V_B \) that results in the transistor operating in saturation, with
\[ I_E = \frac{V_B - 0.7}{1} = V_B - 0.7 \]
\[ I_C = \frac{3 - (V_B - 0.5)}{1} = 3.5 - V_B \]
\[ I_B = I_E - I_C = 2V_B - 4.2 \]
For $\beta_{\text{forced}} = 2$,
\[
\frac{I_C}{I_B} = 2
\]
\[
\frac{3.5 - V_B}{2V_B - 4.2} = 2
\]
\[
\Rightarrow V_B = 2.38 \text{ V}
\]

6.53 Refer to the circuit in Fig. P6.53.

(a) For $V_B = -1 \text{ V}$,
\[
V_E = V_B - V_{BE} = -1 - 0.7 = -1.7 \text{ V}
\]
\[
I_E = \frac{V_E - (-3)}{1} = \frac{-1.7 + 3}{1} = 1.3 \text{ mA}
\]
Assuming active-mode operation, we have
\[
I_C = \alpha I_E \simeq I_E = 1.3 \text{ mA}
\]
\[
V_C = +3 - I_C \times 1 = 3 - 1.3 = +1.7 \text{ V}
\]
Since $V_C > V_B - 0.4$, the transistor is operating in the active mode as assumed.

(b) For $V_B = 0 \text{ V}$,
\[
V_E = 0 - V_{BE} = -0.7 \text{ V}
\]
\[
I_E = \frac{-0.7 - (-3)}{1} = 2.3 \text{ mA}
\]
Assuming operation in the active mode, we have
\[
I_C = \alpha I_E \simeq I_E = 2.3 \text{ mA}
\]
\[
V_C = +3 - I_C \times 1 = 3 - 2.3 = +0.7 \text{ V}
\]
Since $V_C > V_B - 0.4$, the BJT is operating in the active mode, as assumed.

(c) For $V_B = +1 \text{ V}$,
\[
V_E = 1 - 0.7 = +0.3 \text{ V}
\]
\[
I_E = \frac{0.3 - (-3)}{1} = 3.3 \text{ mA}
\]
Assuming operation in the active mode, we have
\[
I_C = \alpha I_E \simeq I_E = 3.3 \text{ mA}
\]
\[
V_C = 3 - 3.3 \times 1 = -0.3 \text{ V}
\]
Now $V_C < V_B - 0.4$, indicating that the transistor is operating in saturation, and our original assumption is incorrect. It follows that
\[
V_C = V_E + V_{CE\text{sat}}
\]
\[
= 0.3 + 0.2 = 0.5 \text{ V}
\]
\[
I_C = \frac{3 - V_C}{1} = \frac{3 - 0.5}{1} = 2.5 \text{ mA}
\]
\[
I_B = I_E - I_C = 3.3 - 2.5 = 0.8 \text{ mA}
\]
\[
\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{2.5}{0.8} = 3.1
\]

(d) When $V_B = 0 \text{ V}$, $I_E = 2.3 \text{ mA}$. The emitter current becomes 0.23 mA at
\[
V_B = -3 + 0.23 \times 1 + 0.7 = -2.07 \text{ V}
\]
(e) The transistor will be at the edge of conduction when $I_E \simeq 0$ and $V_{BE} = 0.5 \text{ V}$, that is,
\[
V_B = -3 + 0.5 = -2.5 \text{ V}
\]
In this case,
\[
V_E = -3 \text{ V}
\]
\[
V_C = +3 \text{ V}
\]

(f) The transistor reaches the edge of saturation when $V_{CE} = 0.3 \text{ V}$ but $I_C = \alpha I_E \simeq I_E$;
\[
V_E = V_B - 0.7
\]
\[
I_E = \frac{V_B - 0.7 - (-3)}{1} = V_B + 2.3
\]
\[
V_C = V_E + 0.3 = V_B - 0.4
\]
\[
I_C = \frac{3 - V_C}{1} = \frac{3 - V_B + 0.4}{1} = 3.4 - V_B
\]
Since
\[
I_C \simeq I_E
\]
\[
3.4 - V_B = V_B + 2.3
\]
\[
V_B = 0.55 \text{ V}
\]
For this value,
\[
V_E = 0.55 - 0.7 = -0.15 \text{ V}
\]
\[
V_C = -0.15 + 0.3 = +0.15 \text{ V}
\]

(g) For the transistor to operate in saturation with $\beta_{\text{forced}} = 2$,
\[
V_E = V_B - 0.7
\]
\[
I_E = \frac{V_B - 0.7 - (-3)}{1} = V_B + 2.3
\]
\[
V_C = V_E + V_{CE\text{sat}} = V_B - 0.7 + 0.2 = V_B - 0.5
\]
\[
I_C = \frac{3 - (V_B - 0.5)}{1} = 3.5 - V_B
\]
\[
I_B = I_E - I_C = 2 V_B - 1.2
\]
\[
I_C = \frac{3.5 - V_B}{2 V_B - 1.2} = 2
\]
\[
\Rightarrow V_B = +1.18 \text{ V}
\]
6.54 (a) $V_B = 0 \text{ V}$

Figure 1

The analysis is shown on the circuit diagram in Fig. 1. The circled numbers indicate the order of the analysis steps.

(b) The transistor cuts off at the value of $V_B$ that causes the 2-mA current of the current source feeding the emitter to flow through the 1-k$\Omega$ resistor connected between the emitter and ground. The circuit under these conditions is shown in Fig. 2.

Observe that $V_E = -2 \text{ mA} \times 1 \text{ k}\Omega = -2 \text{ V}$, $I_E = 0$, and $V_B = V_E + 0.5 = -1.5 \text{ V}$. Since $I_C = 0$, all the 4 mA supplied by the current source feeding the collector flows through the collector 1-k$\Omega$ resistor, resulting in $V_C = +4 \text{ V}$.

Figure 2

(c) Figure 3 shows the transistor at the edge of saturation. Here $V_{CE} = 0.3 \text{ V}$ and $I_C = \alpha I_E \simeq I_E$. A node equation at the emitter gives

$I_E = 2 + V_B - 0.7 = V_B + 1.3 \text{ mA}$

A node equation at the collector gives

$I_C = 4 - (V_B - 0.4) = 4.4 - V_B \text{ mA}$

Imposing the condition $I_C \simeq I_E$ gives

$4.4 - V_B = V_B + 1.3$

$\Rightarrow V_B = +1.55 \text{ V}$

Correspondingly,

$V_E = +0.85 \text{ V}$

$V_C = +1.15 \text{ V}$

6.55

Figure 1
From Fig. 1 we see that
\[ R_1 + R_2 = \frac{V_{CC}}{0.1 \text{ mA}} = \frac{3}{0.1} = 30 \text{ k}\Omega \]
\[ V_{CC} \frac{R_2}{R_1 + R_2} = 1.2 \]
\[ 3 \times \frac{R_2}{30} = 1.2 \]
⇒ \[ R_2 = 12 \text{ k}\Omega \]
\[ R_1 = 30 - 12 = 18 \text{ k}\Omega \]

For \( \beta = 100 \), to obtain the collector current, we replace the voltage divider with its Thévenin equivalent, consisting of
\[ V_{BB} = 3 \times \frac{R_2}{R_1 + R_2} = 3 \times \frac{12}{18 + 12} = 1.2 \text{ V} \]
\[ R_B = R_1 \parallel R_2 = 12 \parallel 18 = 7.2 \text{ k}\Omega \]

Refer to Fig. 2. Assuming active-mode operation, we can write a loop equation for the base–emitter loop:
\[ V_{BB} = I_B R_B + V_{BE} + I_E R_E \]
\[ 1.2 = \frac{I_E}{\beta + 1} \times 7.2 + 0.7 + I_E \times 1 \]
⇒ \[ I_E = \frac{1.2 - 0.7}{1 + \frac{7.2}{101}} = 0.47 \text{ mA} \]
\[ I_C = \alpha I_E = 0.99 \times 0.47 = 0.46 \text{ mA} \]
\[ V_C = +3 - 0.46 \times 1 = +2.54 \text{ V} \]

Since \( V_B = I_E R_B + V_{BE} = 0.47 + 0.7 = 1.17 \text{ V} \), we see that \( V_C > V_B - 0.4 \), and thus the transistor is operating in the active region, as assumed.

---

6.56 Refer to the circuit in Fig. P6.56.
\[ V_E = 1 \text{ V} \]
\[ I_E = \frac{3 - 1}{5} = 0.4 \text{ mA} \]
\[ V_B = V_E - 0.7 = 0.3 \text{ V} \]
\[ I_B = \frac{V_B}{50 \text{ k}\Omega} = \frac{0.3}{50} = 0.006 \text{ mA} \]
\[ I_C = I_E - I_B = 0.4 - 0.006 = 0.394 \text{ mA} \]
\[ V_C = -3 + 5 \times 0.394 = -1.03 \text{ V} \]

Observe that \( V_C < V_B \), confirming our implicit assumption that the transistor is operating in the active region.
\[ \beta = \frac{I_C}{I_B} = \frac{0.394}{0.006} = 66 \]
\[ \alpha = \frac{I_C}{I_E} = \frac{0.394}{0.4} = 0.985 \]

6.57

Refer to the figure. To obtain \( I_E = 0.5 \text{ mA} \) we select \( R_E \) according to
\[ R_E = \frac{3 - 0.7}{0.5} = 4.6 \text{ k}\Omega \]

To obtain \( V_C = -1 \text{ V} \), we select \( R_C \) according to
\[ R_C = \frac{-1 - (-3)}{0.5} = 4 \text{ k}\Omega \]

where we have utilized the fact that \( \alpha \approx 1 \) and thus \( I_C \approx I_E = 0.5 \text{ mA} \). From the table of 5% resistors in Appendix J we select
\[ R_E = 4.7 \text{ k}\Omega \quad \text{and} \quad R_C = 3.9 \text{ k}\Omega \]

For these values,
\[ I_E = \frac{3 - 0.7}{4.7} = 0.49 \text{ mA} \]
\[ I_C \approx I_E = 0.49 \text{ mA} \]
\[ V_{BC} = 0 - V_C = -(3 + 0.49 \times 3.9) = -1.1 \text{ V} \]
Writing a loop equation for the EBJ loop, we have

\[ 3 = I_E R_E + V_{EB} + I_B R_B \]  

(1)

\[ I_E = \frac{3 - 0.7}{2.2 + \frac{20}{41}} \times 0.86 = 0.86 \text{ mA} \]

\[ V_E = 3 - 0.86 \times 2.2 = +1.11 \text{ V} \]

\[ V_B = V_E - 0.7 = +0.41 \text{ V} \]

Assuming active-mode operation, we obtain

\[ I_C = \alpha I_E = \frac{40}{41} \times 0.86 = 0.84 \text{ mA} \]

\[ V_C = -3 + 0.84 \times 2.2 = -1.15 \text{ V} \]

Since \( V_C < V_B + 0.4 \), the transistor is operating in the active mode, as assumed. Now, if \( R_B \) is increased to 100 kΩ, the loop equation [Eq. (1)] yields

\[ I_E = \frac{3 - 0.7}{2.2 + 100} \times \frac{41}{20} = 0.5 \text{ mA} \]

\[ V_E = 3 - 0.5 \times 2.2 = +1.9 \text{ V} \]

\[ V_B = V_E - V_{EB} = 1.9 - 0.7 = +1.2 \text{ V} \]

Assuming active-mode operation, we obtain

\[ I_C = \alpha I_E = \frac{40}{41} \times 0.5 = 0.48 \text{ mA} \]

\[ V_C = -3 + 0.48 \times 2.2 = -1.9 \text{ V} \]

Since \( V_C < V_B + 0.4 \), the transistor is operating in the active mode, as assumed.

If with \( R_B = 100 \text{ kΩ} \), we need the voltages to remain at the values obtained with \( R_B = 20 \text{ kΩ} \), the transistor must have a β value determined as follows. For \( I_E \) to remain unchanged,

\[ \frac{3 - 0.7}{2.2 + \frac{20}{41}} = \frac{3 - 0.7}{2.2 + \frac{100}{\beta + 1}} \]

\[ \Rightarrow \frac{20}{41} = \frac{100}{\beta + 1} \]

\[ \beta + 1 = \frac{410}{2} = 205 \]

\[ \beta = 204 \]

Assume active-mode operation:

\[ I_E = \frac{3 - V_{EB}}{R_E + \frac{R_B}{\beta + 1}} \]

\[ I_E = \frac{3 - 0.7}{2.2 + \frac{20}{51}} = 0.887 \text{ mA} \]

\[ I_B = \frac{I_E}{\beta + 1} = \frac{0.887}{51} = 0.017 \text{ mA} \]

\[ I_C = I_E - I_B = 0.887 - 0.017 = 0.870 \text{ mA} \]

\[ V_B = I_B R_B = 0.017 \times 20 = 0.34 \text{ V} \]

\[ V_E = V_B + V_{EB} = 0.34 + 0.7 = 1.04 \text{ V} \]

\[ V_C = -3 + I_C R_C = -3 + 0.87 \times 2.2 = -1.09 \text{ V} \]

Thus, \( V_C < V_B + 0.4 \), which means active-mode operation, as assumed. The maximum value of \( R_C \) that still guarantees active-mode operation is that which causes \( V_C \) to be 0.4 V above \( V_B \): that is, \( V_C = 0.34 + 0.4 = 0.74 \text{ V} \).

Correspondingly,

\[ R_{C_{\text{max}}} = \frac{0.74 - (-3)}{0.87} = 4.3 \text{ kΩ} \]
A loop equation for the EB loop yields
\[ S = I_E R_B + V_{BE} + I_E R_E \]
\[ \Rightarrow I_E = \frac{5 - 0.7}{R_E + \frac{R_B}{\beta + 1}} \]
\[ I_E = \frac{4.3}{1 + \frac{R_B}{101}} \]
(a) For \( R_B = 100 \, \text{k}\Omega \),
\[ I_E = \frac{4.3}{1 + \frac{100}{101}} = 2.16 \, \text{mA} \]
\[ V_E = I_E R_E = 2.16 \times 1 = 2.16 \, \text{V} \]
\[ V_B = V_E + 0.7 = 2.86 \, \text{V} \]
Assuming active-mode operation, we obtain
\[ I_C = \alpha I_E = 0.99 \times 2.16 = 2.14 \, \text{mA} \]
\[ V_C = 5 - 2.14 \times 1 = 2.86 \, \text{V} \]
Since \( V_C > V_B - 0.4 \), the transistor is operating in the active region, as assumed.
(b) For \( R_B = 10 \, \text{k}\Omega \),
\[ I_E = \frac{4.3}{1 + \frac{10}{101}} = 3.91 \, \text{mA} \]
\[ V_E = 3.91 \times 1 = 3.91 \, \text{V} \]
\[ V_B = 3.91 + 0.7 = 4.61 \, \text{V} \]
Assuming active-mode operation, we obtain
\[ I_C = \alpha I_E = 0.99 \times 3.91 = 3.87 \, \text{mA} \]
\[ V_C = 5 - 3.87 = 1.13 \, \text{V} \]
Since \( V_C < V_B - 0.4 \), the transistor is operating in saturation, contrary to our original assumption. Therefore, we need to redo the analysis assuming saturation-mode operation, as follows:
\[ V_B = V_E + 0.7 \]
\[ V_C = V_E + V_{CE_{sat}} = V_E + 0.2 \]
\[ I_B = \frac{5 - V_B}{R_B} \]
\[ = \frac{5 - 3.7}{10} = 4.3 - V_E \]
\[ I_C = \frac{5 - V_C}{R_C} = 5 - V_E - 0.2 \]
\[ I_E = \frac{V_E}{R_E} = \frac{1}{V_E} = V_E \]
Substituting from Eqs. (1), (2), and (3) into
\[ I_E = I_B + I_C \]
gives
\[ V_E = 0.43 - 0.1 \, V_E + 4.8 - V_E \]
\[ \Rightarrow V_E = 2.5 \, \text{V} \]
\[ V_C = 2.7 \, \text{V} \]
\[ V_B = 3.2 \, \text{V} \]
\[ I_B = \frac{5 - 3.2}{10} = 0.18 \, \text{mA} \]
\[ I_C = \frac{5 - 2.7}{1} = 2.3 \, \text{mA} \]
Thus,
\[ \frac{I_C}{I_B} = 2.3 \, 0.18 = 12.8 \]
which is lower than the value of \( \beta \), verifying saturation-mode operation.
(c) For \( R_B = 1 \, \text{k}\Omega \), we assume saturation-mode operation:
\[ V_B = V_E + 0.7 \]
\[ V_C = V_E + 0.2 \]
\[ I_B = \frac{5 - (V_E + 0.7)}{1} = 4.3 - V_E \]
\[ I_C = \frac{5 - (V_E + 0.2)}{1} = 4.8 - V_E \]
\[ I_E = \frac{V_E}{1} = V_E \]
These values can be substituted into
\[ I_E = I_B + I_C \]
to obtain
\[ V_E = 4.3 - V_E + 4.8 - V_E \]
\[ \Rightarrow V_E = 3 \, \text{V} \]
\[ V_B = 3.7 \, \text{V} \]
\[ V_C = 3.2 \, \text{V} \]
Now checking the currents,
\[ I_B = \frac{5 - 3.7}{1} = 1.3 \, \text{mA} \]
\[ I_C = \frac{5 - 3.7}{1} = 1.8 \, \text{mA} \]
Thus, the transistor is operating at a forced \( \beta \) of
\[ \beta_{forced} = \frac{I_C}{I_B} = \frac{1.8}{1.3} = 1.4 \]
which is much lower than the value of \( \beta \), confirming operation in saturation.
6.61

(a) $I_C = 0.5 \, mA$

$V_2 = 3 - 0.5 \times 3.6 = +1.2 \, V$

$V_1 = -0.7 \, V$

$0.5 \, mA$

(b) $I_B \approx 0 \, mA$

$V_3 = 3 - 0.5 \times 3.6 = +1.2 \, V$

$V_E = -0.7 \, V$

$0.5 \, mA$

(c) $I_B = 0 \, V$

$V_5 = -0.7 \, V$

$V_6 = 0 \, V$

$0.5 \, mA$

(d) $I_B \approx 0 \, V$

$I_E = \frac{3 - 1.45}{6.2} = 0.25 \, mA$

$V_8 = 0.75 + 0.7 = 1.45 \, V$

$0.25 \, mA$

(e) $I = \frac{6}{480} = 0.0125 \, mA$

$V_{10} = -3 + 0.0125 \times 300 = +0.75 \, V$

$0.25 \, mA$

For the solutions and answers to parts (a) through (e), see the corresponding circuit diagrams.
See solution and answer on the figure, which corresponds to Fig. P6.61(a).

See solution and answer on the figure, which corresponds to Fig. P6.61(b).
Writing an equation for the loop containing the BEJ of the transistor leads to
\[ I_E = \frac{3 - 0.7}{4.7 + 101} = 0.449 \text{ mA} \]
\[ V_5 = -3 + 0.449 \times 4.7 = -0.9 \text{ V} \]
\[ V_6 = -0.9 + 0.7 = 0.2 \text{ V} \]
\[ I_C = \alpha I_E = 0.99 \times 0.449 = 0.444 \text{ mA} \]
\[ V_7 = 3 - 0.444 \times 3.6 = 1.4 \text{ V} \]

(d)

An equation for the loop containing the EBJ of the transistor yields
\[ I_E = \frac{3 - 0.75 - 0.7}{6.2 + \frac{110}{101}} = 0.213 \text{ mA} \]
\[ V_8 = +3 - 0.213 \times 6.2 = +1.7 \text{ V} \]
\[ I_C = \alpha I_E = 0.99 \times 0.213 = 0.21 \text{ mA} \]
\[ V_9 = -3 + 0.21 \times 10 = -0.9 \text{ V} \]

(e) See figure on next page.

First, we use Thévenin’s theorem to replace the voltage divider feeding the base with \( V_{BB} \) and \( R_B \):
\[ V_{BB} = -3 + \frac{6}{480} \times 300 = +0.75 \text{ V} \]
\[ R_B = 180 \parallel 300 = 112.5 \text{ k} \Omega \]

Next we write an equation for the loop containing the EBJ to obtain
\[ I_E = \frac{3 - 0.75 - 0.7}{6.2 + \frac{112.5}{101}} = 0.212 \text{ mA} \]
\[ V_{11} = +3 - 0.212 \times 6.2 = +1.7 \text{ V} \]
\[ V_{10} = 1.7 - 0.7 = +1 \text{ V} \]
\[ I_C = \alpha I_E = 0.99 \times 0.212 = 0.21 \text{ mA} \]
\[ V_{12} = -3 + 0.21 \times 10 = -0.9 \text{ V} \]

6.63

We required \( I_E \) to be nominally 1 mA (i.e., at \( \beta = 100 \)) and to remain within ±10% as \( \beta \) varies from 50 to 150. Writing an equation for the loop containing the EBJ results in
\[ I_E = \frac{5 - 0.7}{R_E + \frac{R_B}{\beta + 1}} \]
Thus,
\[ \frac{4.3}{R_E + \frac{R_B}{51}} = 1 \quad (1) \]
\[ \frac{4.3}{R_E + \frac{R_B}{101}} = I_{E_{\text{min}}} \quad (2) \]
This figure belongs to Problem 6.62, part (e).

\[
\frac{4.3}{R_E + \frac{R_B}{15}} = I_{E_{\text{max}}} \tag{3}
\]

If we set \(I_{E_{\text{min}}} = 0.9\) mA and solve Eqs. (1) and (2) simultaneously, we obtain

\[
R_E = 3.81\ \text{k}\Omega
\]

\[
R_B = 49.2\ \text{k}\Omega
\]

Substituting these values in Eqs. (2) and (3) gives

\[
I_{E_{\text{min}}} = 0.9\ \text{mA}
\]

\[
I_{E_{\text{max}}} = 1.04\ \text{mA}
\]

Obviously, this is an acceptable design.

Alternatively, if we set \(I_{E_{\text{max}}} = 1.1\) mA and solve Eqs. (1) and (3) simultaneously, we obtain

\[
R_E = 3.1\ \text{k}\Omega
\]

\[
R_B = 119.2\ \text{k}\Omega
\]

Substituting these values in Eqs. (2) and (3) gives

\[
I_{E_{\text{min}}} = 0.8\ \text{mA}
\]

\[
I_{E_{\text{max}}} = 1.1\ \text{mA}
\]

Obviously this is not an acceptable design (\(I_{E_{\text{min}}} is 20% lower than nominal).)

Therefore, we shall use the first design.

Specifying the resistor values to the nearest kilohm results in

\[
R_E = 4\ \text{k}\Omega
\]

\[
R_B = 50\ \text{k}\Omega
\]

To obtain the value of \(R_C\), we note that at the nominal emitter current value of 1 mA, \(V_C = -1\) V,

\[
I_C = \alpha I_E = 0.99\ \text{mA}
\]

\[
R_C = \frac{-1 - (-5)}{0.99} = 4.04\ \text{k}\Omega
\]

Specified to the nearest kilohm,

\[
R_C = 4\ \text{k}\Omega
\]

Finally, for our design we need to determine the range obtained for collector current and collector voltage for \(\beta\) ranging from 50 to 150 with a nominal value of 100. We compute the nominal value of \(I_E\) from

\[
I_{E_{\text{nominal}}} = \frac{4.3}{4 + \frac{50}{101}} = 0.96\ \text{mA}
\]

We utilize Eqs. (2) and (3) to compute \(I_{E_{\text{min}}} and I_{E_{\text{max}}},

\[
I_{E_{\text{min}}} = \frac{4.3}{4 + \frac{50}{51}} = 0.86\ \text{mA}
\]

\[
I_{E_{\text{max}}} = \frac{4.3}{4 + \frac{50}{151}} = 0.99\ \text{mA}
\]

Thus,

\[
\frac{I_{E_{\text{max}}}}{I_{E_{\text{nominal}}}} = \frac{0.99}{0.96} = 1.03
\]

\[
\frac{I_{E_{\text{min}}}}{I_{E_{\text{nominal}}}} = \frac{0.86}{0.96} = 0.9
\]

which meet our specifications. The collector currents are

\[
I_{C_{\text{nominal}}} = 0.99 \times 0.96 = 0.95\ \text{mA}
\]

\[
I_{C_{\text{min}}} = 0.99 \times 0.86 = 0.85\ \text{mA}
\]

\[
I_{C_{\text{max}}} = 0.99 \times 0.99 = 0.98\ \text{mA}
\]

and the collector voltages are

\[
V_{C_{\text{nominal}}} = -5 + 0.95 \times 4 = -1.2\ \text{V}
\]

\[
V_{C_{\text{min}}} = -5 + 0.85 \times 4 = -1.6\ \text{V}
\]

\[
V_{C_{\text{max}}} = -5 + 0.98 \times 4 = -1.1\ \text{V}
\]
6.64

\[ I_B = \frac{2.3 \text{ V}}{100 \text{ k}\Omega} = 0.023 \text{ mA} \]

Since \( V_C = 2 \text{ V} \) is lower than \( V_B \), which is \(+2.3 \text{ V} \), the transistor will be operating in the active mode. Thus,

\[ I_C = \beta I_B = 50 \times 0.023 = 1.15 \text{ mA} \]

To obtain \( V_C = 2 \text{ V} \), we select \( R_C \) according to

\[ R_C = \frac{V_C}{I_C} = \frac{2 \text{ V}}{1.15 \text{ mA}} = 1.74 \text{ k}\Omega \]

Now, if the transistor is replaced with another having \( \beta = 100 \), then

\[ I_C = 100 \times 0.023 = 2.3 \text{ mA} \]

which would imply

\[ V_C = 2.3 \times 1.74 = 4 \text{ V} \]

which is impossible because the base is at 2.3 V. Thus the transistor must be in the saturation mode and

\[ V_C = V_E - V_{CE_{sat}} \]

\[ = 3 - 0.2 = 2.8 \text{ V} \]

6.65

(a) Consider first the case \( \beta = \infty \) and \( R \)

\[ \begin{align*}
\beta = \infty \text{ and } R \text{ is open circuited. The circuit is shown in Fig. 1, where } \beta = \infty \text{ and } R \text{ is open circuited. Since } \ V_{D1} = V_{BE1}, \text{ we have} \\
V_1 = V_{E1} \\
\text{Thus,} \\
I_{D1} \times 40 = I_{E1} \times 2 \\
\Rightarrow I_{D1} = 0.05I_{E1} \\
\text{But} \\
I_{D1} = \frac{9 - 0.7}{80 + 40} = 0.069 \text{ mA} \approx 0.07 \text{ mA} \\
\text{Thus,} \\
I_{E1} = \frac{0.069}{0.05} = 1.38 \text{ mA} \approx 1.4 \text{ mA} \\
V_{E1} = I_{E1} \times 2 = 2.77 \text{ V} \approx 2.8 \text{ V} \\
V_{B1} = V_{E1} + 0.7 = 3.5 \text{ V} \\
I_{C1} = I_{E1} = 1.38 \text{ mA} \approx 1.4 \text{ mA} \\
V_2 = 9 - I_{C1} \times 2 = 9 - 1.38 \times 2 \approx 6.2 \text{ V} \\
V_{C1} = V_2 - V_{D2} = 6.2 - 0.7 = 5.5 \text{ V} \\
V_{E2} = V_2 = 6.2 \text{ V} \\
I_{E2} = \frac{9 - 6.2}{100 \text{ } \Omega} = 28 \text{ mA} \\
I_{C2} = I_E = 28 \text{ mA} \\
V_{C2} = 28 \times 0.1 = 2.8 \text{ V}
\end{align*} \]
Now connecting the resistance \( R = 2 \, k\Omega \) between \( C_1 \) and \( E_2 \) (see Fig. 2) both of which at 2.8 V, will result in zero current through \( R \); thus all voltages and currents remain unchanged.

(b) We next consider the situation with \( \beta = 100 \), first with \( R \) disconnected. The circuit is shown in Fig. 3.

Once again we observe that \( V_{E1} = V_1 \), thus

\[
I_{E1} \times 2 = I_{D1} \times 40
\]

\[ \Rightarrow I_{D1} = 0.05I_{E1} \]

The base current of \( Q_1 \) is \( I_{E1}/101 \approx 0.01I_{E1} \). Thus, the current through the 80-k\( \Omega \) resistor is

\[
0.06I_{E1} = \frac{9 - V_{B1}}{80} = \frac{9 - (2I_{E1} + 0.7)}{80}
\]

\[ \Rightarrow I_{E1} = 1.22 \, mA \]

\[ V_{E1} = 1.22 \times 2 = 2.44 \, V \]

\[ V_{B1} = 2.44 + 0.7 = 3.14 \, V \]

Thus,

\[
I_{C1} = \alpha I_{E1} = 0.99 \times 1.22 = 1.21 \, mA
\]

Observing that \( V_{E2} = V_2 \), we see that the voltage drops across the 2-k\( \Omega \) resistor and the 100-\( \Omega \) resistor are equal, thus

\[
I_{D2} \times 2 = I_{E2} \times 0.1
\]

\[ \Rightarrow I_{D2} = 0.05I_{E2} \]

As the base current of \( Q_2 \) is approximately \( 0.01I_{E2} \), a node equation at \( C_1 \) yields

\[
I_{D2} = I_{E1} - 0.01I_{E2}
\]

Finally, with the resistance \( R \) connected between \( E_1 \) and \( C_2 \), it will conduct a current that we can initially estimate as

\[
I = \frac{V_{E1} - V_{C2}}{R} = \frac{2.44 - 2}{2} = 0.22 \, mA
\]

This is a substantial amount compared to \( I_{E1} = 1.22 \, mA \), requiring that we redo the analysis with \( R \) in place. The resulting circuit is shown in Fig. 4.
Denoting the emitter voltage of $Q_1$, $V_{E1}$, and the current through $R$ as $I$, the analysis proceeds as follows:

- $V_1 = V_{E1}$
- $I_{D1} = \frac{V_{E1}}{40} = 0.025V_{E1}$
- $I_{E1} = \frac{V_{E1}}{2} + I = 0.5V_{E1} + I$
- $I_{R1} = \frac{I_{E1}}{100} = 0.005V_{E1} + 0.01I$
- $V_{C2} = V_{E1} - I \times 2 = V_{E1} - 2I$
- $I_{C2} = -I + \frac{V_{C2}}{0.1} = 10V_{E1} - 21I$
- $I_{E2} = \frac{I_{C2}}{\alpha} = 10.1V_{E1} - 21.2I$
- $I_{D2} = I_{C1} - I_{R2} = 0.395V_{E1} + 1.2I$

Substituting $I = 0.05V_{E2}$ gives

- $V_{E1} = 2.41 V$
- $I = 0.12 mA$

Substituting these quantities in the equations above gives

- $V_{B1} = 2.41 + 0.7 = 3.11 V$

$V_{E2} = 6.82 V$

$V_{C2} = 2.17 V$

**6.66 (a) $\beta = \infty$**

The analysis and results are given in the circuit diagram in Fig. 1 on page 27. The circled numbers indicate the order of the analysis steps.

**6.66 (b) $\beta = 100$**

- $I_{E1} = 1.325 mA$
- $I_{C1} = 1.31 mA$
- $I_{D1} = 1.09 mA$
- $V_{C1} = 9 - 1.09 \times 2 - 0.7 = 6.12 V$
- $V_{E2} = 6.82 V$
- $I_{E2} = 9 - 6.82 = 21.8 mA$
- $I_{C2} = 0.99 \times 21.8 = 21.6 mA$
- $V_{C2} = 2.17 V$

![Figure 4](image-url)
This figure belongs to Problem 6.66, part (a).

By reference to Fig. 2 on page 26, we can write an equation for the loop containing the EBJ of \( Q_1 \) as follows:

\[ 3 = I_{E1} \times 9.1 + 0.7 + I_{B1} \times 100 \]

Substituting \( I_{B1} = I_{E}/(\beta + 1) = I_{E}/101 \) and rearranging, we obtain

\[ I_{E1} = \frac{3 - 0.7}{9.1 + 100} = 0.228 \text{ mA} \]

Thus,

\[ I_{B1} = \frac{I_{E1}}{101} = 0.0023 \text{ mA} \]
\[ V_2 = V_1 + 0.7 = 0.93 \text{ V} \]
\[ I_{C1} = \alpha I_{E1} = 0.99 \times 0.228 = 0.226 \text{ mA} \]

Then we write a node equation at \( C_1 \):

\[ I_{C1} = I_{E2} + \frac{V_3 - (-3)}{9.1} \]

Substituting for \( I_{C1} = 0.226 \text{ mA}, I_{E2} = I_{E2}/101, \)
and \( V_3 = V_4 + 0.7 = -3 + I_{E2} \times 4.3 + 0.7 \) gives

\[ 0.226 = \frac{I_{E2}}{101} + \frac{-3 + 4.3 I_{E2} + 0.7 + 3}{9.1} \]

\[ I_{E2} = \frac{9.1 I_{E2} + 4.3 I_{E2} + 0.7 + 3 \times 101}{9.1} \]

\[ \Rightarrow I_{E2} = 0.31 \text{ mA} \]

\[ I_{C1} - I_{E2} = 0.226 - 0.0031 = 0.223 \text{ mA} \]
\[ V_3 = -3 + 0.223 \times 9.1 = -0.97 \text{ V} \]
\[ V_4 = V_3 - 0.7 = -1.7 \text{ V} \]

The circuit with the selected resistor values is shown in Fig. 2. Analysis of the circuit proceeds as follows:

\[ V_2 = -0.7 \text{ V} \]
This figure belongs to Problem 6.67.

![Figure 1](image1)

This figure belongs to Problem 6.67.

![Figure 2](image2)

\[ I_{E1} = \frac{V_2 - (-5)}{8.2} = \frac{-0.7 + 5}{8.2} = 0.524 \text{ mA} \]

\[ I_{C1} = \alpha I_{E1} = 0.99 \times 0.524 = 0.52 \text{ mA} \]

The current \( I_1 \) through the 10-k\( \Omega \) resistor is given by

\[ I_1 = I_{C1} - I_{E1} = I_{E2} - \frac{I_{E2}}{101} \]

Noting that the voltage drop across the 10 k\( \Omega \) resistor is equal to \( (0.7 + I_{E2} \times 8.2) \), we can write

\[ I_1 \times 10 = 8.2I_{E2} + 0.7 \]

Thus,

\[ 10 \left( 0.52 - \frac{I_{E2}}{101} \right) = 8.2I_{E2} + 0.7 \]

\( \Rightarrow I_{E2} = 0.542 \text{ mA} \)

\[ V_4 = 5 - 0.542 \times 8.2 = 0.56 \text{ V} \]

\[ V_3 = 0.56 - 0.7 = -0.14 \text{ V} \]

\[ I_{C2} = \alpha I_{E2} = 0.99 \times 0.542 = 0.537 \text{ mA} \]

\[ I_2 = I_{C2} - I_{B3} = 0.537 - \frac{I_{E3}}{101} \]

Since the voltage drop across the 6.2-k\( \Omega \) resistor is equal to \( (0.7 + I_{E3} \times 2.4) \),

\[ I_2 \times 6.2 = 0.7 + 2.4I_{E3} \]

\[ 6.2 \left( 0.537 - \frac{I_{E3}}{101} \right) = 0.7 + 2.4I_{E3} \]

\( \Rightarrow I_{E3} = 1.07 \text{ mA} \)
\[ V_6 = -5 + 1.07 \times 2.4 = -2.43 \text{ V} \]
\[ V_5 = V_6 + 0.7 = -1.73 \]
\[ I_{E3} = \alpha \times I_{E3} = 0.99 \times 1.07 = 1.06 \text{ mA} \]
\[ V_7 = -3.9 \times 1.06 = 0.87 \text{ V} \]

6.68 Refer to the circuit in Fig. P6.68.

(a) For \( v_I = 0 \), both transistors are cut off and all currents are zero. Thus
\[ V_B = 0 \text{ V} \quad \text{and} \quad V_E = 0 \text{ V} \]

(b) For \( v_I = +2 \text{ V} \), \( Q_1 \) will be conducting and \( Q_2 \) will be cut off, and the circuit reduces to that in Fig. 1.

Since \( V_B \) will be lower than +2 V, \( V_C \) will be higher than \( V_B \) and the transistor will be operating in the active mode. Thus,
\[ I_e = \frac{2 - 0.7}{1 + \frac{10}{51}} = 1.1 \text{ mA} \]
\[ V_E = +1.1 \text{ V} \]
\[ V_B = 1.8 \text{ V} \]

(c) For \( v_I = -2.5 \), \( Q_1 \) will be off and \( Q_2 \) will be on, and the circuit reduces to that in Fig. 2.

\[ I_B = -3 - (-5) = 2.3 \text{ mA} \]
\[ I_C = I_E - I_B = 2.3 - 0.2 = 2.1 \text{ mA} \]
\[ \beta_{saturated} = \frac{I_C}{I_B} = \frac{2.1}{0.2} = 10.5 \]

which is lower than \( \beta \), verifying that \( Q_2 \) is operating in saturation.

Since \( V_B > -2.5 \), \( V_C \) will be lower than \( V_B \) and \( Q_2 \) will be operating in the active region. Thus
\[ I_e = \frac{2.5 - 0.7}{1 + \frac{10}{51}} = 1.5 \text{ mA} \]
\[ V_E = -I_E \times 1 = -1.5 \text{ V} \]
\[ V_B = -1.5 - 0.7 = -2.2 \text{ V} \]

(d) For \( v_I = -5 \text{ V} \), \( Q_1 \) will be off and \( Q_2 \) will be on, and the circuit reduces to that in Fig. 3.
Assuming saturation-mode operation, the terminal voltages are interrelated as shown in the figure, which corresponds to Fig. P6.69(a). Thus we can write

\[ I_E = \frac{V_E}{1} = V_E \]

\[ I_C = \frac{5 - (V_E + 0.2)}{10} = 0.5 - 0.1(V_E + 0.2) \]

\[ I_B = \frac{5 - (V_E + 0.7)}{20} = 0.25 - 0.05(V_E + 0.7) \]

Now, imposing the constraint

\[ I_E = I_C + I_B \]

results in

\[ V_E = 0.5 - 0.1(V_E + 0.2) + 0.25 - 0.05(V_E + 0.7) \]

\[ \Rightarrow V_E = 0.6 \text{ V} \]

\[ V_C = 0.8 \text{ V} \]

\[ V_B = 1.3 \text{ V} \]

\[ I_C = \frac{5 - 0.8}{10} = 0.42 \text{ mA} \]

\[ I_B = \frac{5 - 1.3}{20} = 0.185 \text{ mA} \]

\[ \beta_{\text{forced}} = \frac{0.42}{0.185} = 2.3 \]

which is less than the value of \( \beta_1 \) verifying saturation-mode operation.

Assuming saturation-mode operation, the terminal voltages are interrelated as shown in the figure, which corresponds to Fig. P6.69(b). We can obtain the currents as follows:

\[ I_E = \frac{5 - V_E}{1} = 5 - V_E \]

\[ I_C = \frac{V_E - 0.2}{1} = V_E - 0.2 \]

\[ I_B = \frac{V_E - 0.7 - (-5)}{10} = 0.1 V_E + 0.43 \]

Imposing the constraint

\[ I_E = I_B + I_C \]

results in

\[ 5 - V_E = V_E - 0.2 + 0.1 V_E + 0.43 \]

\[ \Rightarrow V_E = +2.27 \text{ V} \]

\[ V_C = +2.07 \text{ V} \]

\[ V_B = 1.57 \text{ V} \]

\[ I_C = \frac{2.07}{1} = 2.07 \text{ mA} \]

\[ I_B = \frac{1.57 - (-5)}{10} = 0.657 \text{ mA} \]

\[ \beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{2.07}{0.657} = 3.2 \]

which is lower than the value of \( \beta \), verifying saturation-mode operation.
(c) We shall assume that both $Q_3$ and $Q_4$ are operating in saturation. To begin the analysis shown in the figure, which corresponds to Fig. P6.69(c), we denote the voltage at the emitter of $Q_3$ as $V$ and then obtain the voltages at all other nodes in terms of $V$, utilizing the fact that a saturated transistor has $|V_{CE}| = 0.2$ V and of course $|V_{BE}| = 0.7$ V. Note that the choice of the collector node to begin the analysis is arbitrary; we could have selected any other node and denoted its voltage as $V$. We next draw a circle around the two transistors to define a “supernode.” A node equation for the supernode will be

$$IE_3 + IC_4 = IB_3 + I + IE_4$$  \hspace{1cm} (1)

where

$$IE_3 = \frac{5 - (V + 0.2)}{10} = 0.48 - 0.1V$$  \hspace{1cm} (2)

$$IC_4 = \frac{5 - (V - 0.5)}{30} = 0.183 - 0.033V$$  \hspace{1cm} (3)

$$IB_3 = \frac{5 - (V - 0.5)}{10} = 0.1V - 0.05$$  \hspace{1cm} (4)

$$I = \frac{V}{20} = 0.05V$$  \hspace{1cm} (5)

$$IE_4 = \frac{V - 0.7}{10} = 0.1V - 0.07$$  \hspace{1cm} (6)

Substituting from Eqs. (2)–(6) into Eq. (1) gives

$$0.48 - 0.1V + 0.183 - 0.033V = 0.1V - 0.05 + 0.05V + 0.1V - 0.07$$

$$\Rightarrow V = 2.044 \text{ V}$$

Thus

$$V_{C3} = V = 2.044 \text{ V}$$

$$V_{C4} = V - 0.5 = 1.54 \text{ V}$$

Next we determine all currents utilizing Eqs. (2)–(6):

$$IE_3 = 0.276 \text{ mA}$$

$$IC_4 = 0.116 \text{ mA}$$

$$IB_3 = 0.154 \text{ mA}$$

$$I = 0.102 \text{ mA}$$

$$IE_4 = 0.134$$

The base current of $Q_4$ can be obtained from

$$IB_4 = IE_4 - IC_4 = 0.134 - 0.116 = 0.018 \text{ mA}$$

Finally, the collector current of $Q_3$ can be found as

$$IC_3 = I + IB_4 = 0.102 + 0.018 = 0.120$$

The forced $\beta$ values can now be found as

$$\beta_{forced3} = \frac{IC_3}{IB_3} = \frac{0.120}{0.154} = 0.8$$

$$\beta_{forced4} = \frac{IC_4}{IB_4} = \frac{0.116}{0.018} = 6.4$$

Both $\beta_{forced}$ values are well below the $\beta$ value of 50, verifying that $Q_3$ and $Q_4$ are in deep saturation.